Name ____

Date: November 7, 2019 Teacher: Asher Roberts

Multivariable Calculus Exam 1

Vectors and the Geometry of Space
Vector Functions
Time—55 minutes
Number of questions—10

A GRAPHING CALCULATOR MAY BE REQUIRED FOR SOME QUESTIONS ON THIS EXAM.

Directions: Solve each of the following problems, using the available space to show all relevant work. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 10 points. Use separate paper for scratch work, and do not turn in scratch work with your exam. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score			
Question	**Points**	**Question**	**Points**
1		6	
2		7	
3		8	
4		9	
5		10	
Extra Credit:			
Overall Score:			

1. Describe in words the region of $\mathbb{R}^3$ represented by $z^2 = 1$.

2. Find a vector that has the same direction as $\langle -2, 4, 2 \rangle$ but has length 6.

3. Determine whether the vectors $\mathbf{u} = \langle a, b, c \rangle$ and $\mathbf{v} = \langle -b, a, 0 \rangle$ are orthogonal, parallel, or neither.

4. Find the volume of the parallelepiped with vertices $(0, 1, 0)$, $(2, 2, 2)$, $(0, 3, 0)$, and $(3, 1, 2)$.

5. Find an equation of the plane that passes through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$.

6. Use traces to sketch and identify the surface $4x^2 - 16y^2 + z^2 = 16$.

7. Sketch the curve with vector equation $\mathbf{r}(t) = \langle 1, \cos t, 2\sin t\rangle$. Indicate with an arrow the direction in which t increases.

8. Find parametric equations for the tangent line to the curve $x = 7t^2 - 4$, $y = 7t^2 + 4$, $z = 6t + 5$ at the point $(3, 11, 11)$.

9. Find the curvature of $\mathbf{r}(t) = 3t\mathbf{i} + 4\sin t\mathbf{j} + 4\cos t\mathbf{k}$.

10. Find the velocity and position vectors of a particle that has acceleration $\mathbf{a}(t) = 2\mathbf{i} + 6t\mathbf{j} + 12t^2\mathbf{k}$, initial velocity $\mathbf{v}(0) = \mathbf{i}$, and initial position $\mathbf{r}(0) = \mathbf{j} - \mathbf{k}$.

11. (EXTRA CREDIT). Show that the curvature of a curve does not depend on the parametrization of the curve.