Name ____

Date: May 11, 2020 Teacher: Asher Roberts

Multivariable Calculus Exam 3

Multiple Integrals
Time—55 minutes
Number of questions—10

A GRAPHING CALCULATOR MAY BE REQUIRED FOR SOME QUESTIONS ON THIS EXAM.

Directions: Solve each of the following problems, using the available space to show all relevant work. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 10 points. Use separate paper for scratch work, and do not turn in scratch work with your exam. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score			
Question	**Points**	**Question**	**Points**
1		6	
2		7	
3		8	
4		9	
5		10	10
Overall Score:			

1. Evaluate the double integral

$$\iint_R \sqrt{2}\, dA, \quad R = \{(x, y) \mid 2 \leq x \leq 6, -1 \leq y \leq 5\}$$

by first identifying it as the volume of a solid.

2. Evaluate the double integral

$$\iint_D \frac{y}{x^2+1}\, dA, \quad D = \{(x, y) \mid 0 \le x \le 4, 0 \le y \le \sqrt{x}\}.$$

3. Evaluate the integral $\iint_R (2x - y)\, dA$, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = x$, by changing to polar coordinates.

4. Find the mass and center of mass of the lamina that occupies the region

$$D = \{(x, y) \mid 1 \leq x \leq 3, 1 \leq y \leq 4\}$$

and has density function $\rho(x, y) = ky^2$.

5. Find the area of the surface consisting of the part of the plane $6x + 4y + 2z = 1$ that lies inside the cylinder $x^2 + y^2 = 25$.

6. Evaluate the triple integral $\iiint_E y \, dV$, where

$$E = \{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y\}.$$

7. Use cylindrical coordinates to evaluate $\iiint_E \sqrt{x^2 + y^2}\, dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

8. Use spherical coordinates to evaluate $\iiint_B (x^2 + y^2 + z^2)^2 \, dV$, where B is the ball with center the origin and radius 5.

9. Use the transformation $x = 2u$, $y = 3v$ to evaluate the integral $\iint_R x^2 \, dA$, where R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$.

10. Evaluate $\int x^4 e^{-x}\, dx$.