Multivariable Calculus Homework #1

Replace this text with your name

Due: Replace this text with a due date

Exercise (12.1.21). (a) Prove that the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

(b) Find the lengths of the medians of the triangle with vertices $A(1, 2, 3)$, $B(-2, 0, 5)$, and $C(4, 1, 5)$. (A *median* of a triangle is a line segment that joins a vertex to the midpoint of the opposite side.)

Solution: Replace this text with your solution. □

Exercise (12.1.23). Find equations of the spheres with center $(2, -3, 6)$ that touch (a) the xy-plane, (b) the yz-plane, (c) the xz-plane.

Solution: Replace this text with your solution. □

Exercise (12.1.38). Describe in words the region of $\mathbb{R}^3$ represented by the inequality $x^2 + y^2 + z^2 > 2z$.

Solution: Replace this text with your solution. □

Exercise (12.1.41). Write an inequality to describe the region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$.

Solution: Replace this text with your solution. □

Exercise (12.1.46). Find the volume of the solid that lies inside both of the spheres

$$x^2 + y^2 + z^2 + 4x - 2y + 4z + 5 = 0$$

and

$$x^2 + y^2 + z^2 = 4.$$

Solution: Replace this text with your solution. □

Exercise (12.2.21)**.** If $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$, find $\mathbf{a} + \mathbf{b}$, $4\mathbf{a} + 2\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$.

Solution: Replace this text with your solution. □

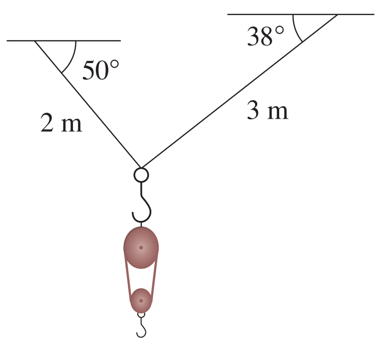
Exercise (12.2.26)**.** Find the vector that has the same direction as $\langle 6, 2, -3\rangle$ but has length 4.

Solution: Replace this text with your solution. □

Exercise (12.2.29)**.** If $\mathbf{v}$ lies in the first quadrant and makes an angle $\pi/3$ with the positive x-axis and $|\mathbf{v}| = 4$, find $\mathbf{v}$ in component form.

Solution: Replace this text with your solution. □

Exercise (12.2.37)**.** A block-and-tackle pulley hoist is suspended in a warehouse by ropes of lengths 2 m and 3 m. The hoist weighs 350 N. The ropes, fastened at different heights, make angles of 50° and 38° with the horizontal. Find the tension in each rope and the magnitude of each tension.



Solution: Replace this text with your solution. □

Exercise (12.2.43)**.** If A, B, and C are the vertices of a triangle, find

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}.$$

Solution: Replace this text with your solution. □

Exercise (12.3.26)**.** Find the values of x such that the angle between the vectors $\langle 2, 1, -1\rangle$, and $\langle 1, x, 0\rangle$ is $45°$.

Solution: Replace this text with your solution. □

Exercise (12.3.45)**.** Show that the vector $\operatorname{orth}_{\mathbf{a}} \mathbf{b} = \mathbf{b} - \operatorname{proj}_{\mathbf{a}} \mathbf{b}$ is orthogonal to $\mathbf{a}$. (It is called an <u>orthogonal projection</u> of $\mathbf{b}$.)

Solution: Replace this text with your solution. □

Exercise (12.3.53)**.** Use scalar projection to show that the distance from a point $P_1(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Solution: Replace this text with your solution. □

Exercise (12.3.55)**.** Find the angle between a diagonal of a cube and one of its edges.

Solution: Replace this text with your solution. □

Exercise (12.3.61)**.** Use Theorem 12.3.2 to prove the Cauchy-Schwarz Inequality:

$$|\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|.$$

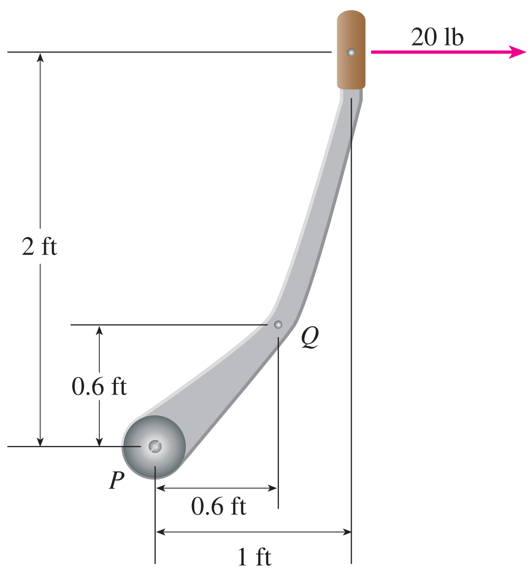
Solution: Replace this text with your solution. □

Exercise (12.4.37)**.** Use the scalar triple product to verify that the vectors $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$, and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ are coplanar.

Solution: Replace this text with your solution. □

Exercise (12.4.40)**.** (a) A horizontal force of 20 lb is applied to the handle of a gearshift lever as shown. Find the magnitude of the torque about the pivot point P.

(b) Find the magnitude of the torque about P if the same force is applied at the elbow Q of the lever.



Solution: Replace this text with your solution. □

Exercise (12.4.43)**.** If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

Solution: Replace this text with your solution. □

Exercise (12.4.45)**.** (a) Let P be a point not on the line L that passes through the points Q and R. Show that the distance d from the point P to the line L is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

where $\mathbf{a} = \overrightarrow{QR}$ and $\mathbf{b} = \overrightarrow{QP}$.

(b) Use the formula in part (a) to find the distance from the point $P(1, 1, 1)$ to the line through $Q(0, 6, 8)$ and $R(-1, 4, 7)$.

Solution: Replace this text with your solution. □

Exercise (12.4.52)**.** Prove that

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = \begin{vmatrix} \mathbf{a} \cdot \mathbf{c} & \mathbf{b} \cdot \mathbf{c} \\ \mathbf{a} \cdot \mathbf{d} & \mathbf{b} \cdot \mathbf{d} \end{vmatrix}.$$

Solution: Replace this text with your solution. □

Exercise (12.5.50)**.** Find the cosine of the angle between the planes $x+y+z=0$ and $x+2y+3z=1$.

Solution: Replace this text with your solution. ☐

Exercise (12.5.63)**.** Find an equation of the plane with x-intercept a, y-intercept b, and z-intercept c.

Solution: Replace this text with your solution. ☐

Exercise (12.5.68)**.** Which of the following four lines are parallel? Are any of them identical?

$$
\begin{aligned}
&L_1 : x = 1 + 6t, \quad y = 1 - 3t, \quad z = 12t + 5 \\
&L_2 : x = 1 + 2t, \quad y = t, \quad z = 1 + 4t \\
&L_3 : 2x - 2 = 4 - 4y = z + 1 \\
&L_4 : \mathbf{r} = \langle 3, 1, 5 \rangle + t\langle 4, 2, 8 \rangle
\end{aligned}
$$

Solution: Replace this text with your solution. ☐

Exercise (12.5.75)**.** Show that the distance between the parallel planes $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$ is

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}.$$

Solution: Replace this text with your solution. ☐

Exercise (12.5.77)**.** Show that the lines with symmetric equations $x=y=z$ and $x+1=y/2=z/3$ are skew, and find the distance between these lines.

Solution: Replace this text with your solution. ☐

Exercise (12.6.42)**.** Use a computer with three-dimensional graphing software to graph the surface $x^2 - 6x + 4y^2 - z = 0$. Experiment with viewpoints and with domains for the variables until you get a good view of the surface.

Solution: Replace this text with your solution. □

Exercise (12.6.46)**.** Find an equation for the surface obtained by rotating the line $z = 2y$ about the z-axis.

Solution: Replace this text with your solution. □

Exercise (12.6.48)**.** Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.

Solution: Replace this text with your solution. □

Exercise (12.6.50)**.** A cooling tower for a nuclear reactor is to be constructed in the shape of a hyperboloid of one sheet. The diameter at the base is 280 m and the minimum diameter, 500 m above the base, is 200 m. Find an equation for the tower.

Solution: Replace this text with your solution. □

Exercise (12.6.52)**.** Show that the curve of intersection of the surfaces $x^2 + 2y^2 - z^2 + 3x = 1$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$ lies in a plane.

Solution: Replace this text with your solution. □