Multivariable Calculus Homework #2

Replace this text with your name

Due: Replace this text with a due date

Exercise (13.1.20)**.** Find a vector equation and parametric equations for the line segment that joins $P(a, b, c)$ to $Q(u, v, w)$.

Solution: Replace this text with your solution. □

Exercise (13.1.32)**.** At what points does the helix $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

Solution: Replace this text with your solution. □

Exercise (13.1.37)**.** Use a computer to graph the curve $\mathbf{r}(t) = \langle \cos 2t, \cos 3t, \cos 4t \rangle$. Make sure you choose a parameter domain and viewpoints that reveal the true nature of the curve.

Solution: Replace this text with your solution. □

Exercise (13.1.45)**.** Find a vector function that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

Solution: Replace this text with your solution. □

Exercise (13.1.49)**.** If two objects travel through space along two different curves, it's often important to know whether they will collide. (Will a missile hit its moving target? Will two aircraft collide?) The curves might intersect, but we need to know whether the objects are in the same position *at the same time*. Suppose the trajectories of two particles are given by the vector functions

$$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle \qquad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$$

for $t \geq 0$. Do the particles collide?

Solution: Replace this text with your solution. □

Exercise (13.2.22)**.** If $\mathbf{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t} \rangle$, find $\mathbf{T}(0)$, $\mathbf{r}''(0)$, and $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$.

Solution: Replace this text with your solution. □

Exercise (13.2.25)**.** Find parametric equations for the tangent line to the curve with parametric equations $x = e^{-t} \cos t$, $y = e^{-t} \sin t$, $z = e^{-t}$ at $(1, 0, 1)$.

Solution: Replace this text with your solution. □

Exercise (13.2.49)**.** Find $f'(2)$, where $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$, $\mathbf{u}(2) = \langle 1, 2, -1 \rangle$, $\mathbf{u}'(2) = \langle 3, 0, 4 \rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3 \rangle$.

Solution: Replace this text with your solution. □

Exercise (13.2.54)**.** Find an expression for $\frac{d}{dt}[\mathbf{u}(t) \cdot (\mathbf{v}(t) \times \mathbf{w}(t))]$.

Solution: Replace this text with your solution. □

Exercise (13.2.57)**.** If $\mathbf{u}(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}''(t)]$, show that

$$\mathbf{u}'(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}'''(t)].$$

Solution: Replace this text with your solution. □

Exercise (13.3.56)**.** Show that the osculating plane at every point on the curve $\mathbf{r}(t) = \langle t+2, 1-t, \frac{1}{2}t^2 \rangle$ is the same plane. What can you conclude about the curve?

Solution: Replace this text with your solution. □

Exercise (13.3.59)**.** Show that the curvature κ is related to the tangent and normal vectors by the equation

$$\frac{d\mathbf{T}}{ds} = \kappa \mathbf{N}.$$

Solution: Replace this text with your solution. □

Exercise (13.3.61)**.** (a) Show that $d\mathbf{B}/ds$ is perpendicular to $\mathbf{B}$.

(b) Show that $d\mathbf{B}/ds$ is perpendicular to $\mathbf{T}$.

(c) Deduce from parts (a) and (b) that $d\mathbf{B}/ds = -\tau(s)\mathbf{N}$ for some number $\tau(s)$ called the torsion of the curve. (The torsion measures the degree of twisting of a curve.)

(d) Show that for a plane curve the torsion is $\tau(s) = 0$.

Solution: Replace this text with your solution. □

Exercise (13.3.62)**.** The following formulas, called the Frenet-Serret formulas, are of fundamental importance in differential geometry:

1. $d\mathbf{T}/ds = \kappa\mathbf{N}$

2. $d\mathbf{N}/ds = -\kappa\mathbf{T} + \tau\mathbf{B}$

3. $d\mathbf{B}/ds = -\tau\mathbf{N}$

Use the fact that $\mathbf{N} = \mathbf{B} \times \mathbf{T}$ to deduce Formula 2 from Formula 1 and 3.

Solution: Replace this text with your solution. □

Exercise (13.3.67)**.** The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 Å $= 10^{-8}$ cm). Each helix rises about 34 Å during each complete turn, and there are about 2.9×10^8 complete turns. Estimate the length of each helix.

Solution: Replace this text with your solution. □

Exercise (13.4.25)**.** A ball is thrown at an angle of $45°$ to the ground. If the ball lands 90 m away, what was the initial speed of the ball?

Solution: Replace this text with your solution. □

Exercise (13.4.29)**.** A medieval city has the shape of a square and is protected by walls with length 500 m and height 15 m. You are the commander of an attacking army and the closest you can get to the wall is 100 m. Your plan is to set fire to the city by catapulting heated rocks over the wall (with an initial speed of 80 m/s). At what range of angles should you tell your men to set the catapult? (Assume the path of the rocks is perpendicular to the wall.)

Solution: Replace this text with your solution. □

Exercise (13.4.35)**.** A particle has position function $\mathbf{r}(t)$. If $\mathbf{r}'(t) = \mathbf{c} \times \mathbf{r}(t)$, where $\mathbf{c}$ is a constant vector, describe the path of the particle.

Solution: Replace this text with your solution. □

Exercise (13.4.44)**.** If a particle with mass m moves with position vector $\mathbf{r}(t)$, then its <u>angular momentum</u> is defined as $\mathbf{L}(t) = m\mathbf{r}(t) \times \mathbf{v}(t)$ and its <u>torque</u> as $\boldsymbol{\tau}(t) = m\mathbf{r}(t) \times \mathbf{a}(t)$. Show that $\mathbf{L}'(t) = \boldsymbol{\tau}(t)$. Deduce that if $\boldsymbol{\tau}(t) = \mathbf{0}$ for all t, then $\mathbf{L}(t)$ is constant. (This is the *law of conservation of angular momentum.*)

Solution: Replace this text with your solution. □

Exercise (13.4.45)**.** The position function of a spaceship is

$$\mathbf{r}(t) = (3+t)\mathbf{i} + (2 + \ln t)\mathbf{j} + \left(7 - \frac{4}{t^2+1}\right)\mathbf{k}$$

and the coordinates of a space station are $(6, 4, 9)$. The captain wants the spaceship to coast into the space station. When should the engines be turned off?

Solution: Replace this text with your solution. □