Name ____

Date: February 26, 2019 Teacher: Asher Roberts

AP Calculus BC Exam 3 Answer Sheet

Example: (A) (B) ● (D) (E)

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)
16. (A) (B) (C) (D) (E)

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

<table>
<tr><th colspan="3">Exam Score</th></tr>
<tr><th>Part</th><th>Number of Correct Answers</th><th>Number of Partially Correct Answers</th></tr>
<tr><td>A</td><td></td><td></td></tr>
<tr><td>B</td><td></td><td></td></tr>
<tr><td>Total:</td><td></td><td></td></tr>
<tr><td></td><td>Overall Score:</td><td></td></tr>
</table>

1. If $\dfrac{dy}{dx} = \tan x$, then $y =$

   (A) $\dfrac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

   (D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

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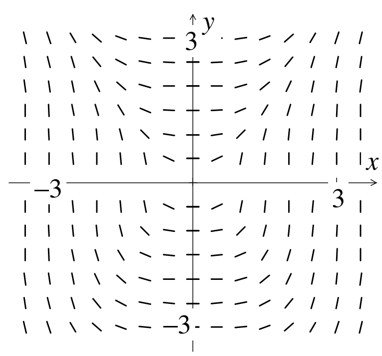
2. $\displaystyle\int_0^1 \left(4 - x^2\right)^{-\frac{3}{2}}\, dx =$

   (A) $\dfrac{2-\sqrt{3}}{3}$ (B) $\dfrac{2\sqrt{3}-3}{4}$ (C) $\dfrac{\sqrt{3}}{12}$ (D) $\dfrac{\sqrt{3}}{3}$ (E) $\dfrac{\sqrt{3}}{2}$

3. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$



4. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

5. If $\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$, then $f(x) =$

(A) $2\sin x + 2x\cos x + C$
(B) $x^2 \sin x + C$
(C) $2x\cos x - x^2 \sin x + C$
(D) $4\cos x - 2x\sin x + C$
(E) $\left(2 - x^2\right)\cos x - 4\sin x + C$

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6. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x} dx$
(B) $\int_a^b \sqrt{x + \tan x} dx$
(C) $\int_a^b \sqrt{1 + \sec^2 x} dx$
(D) $\int_a^b \sqrt{1 + \tan^2 x} dx$
(E) $\int_a^b \sqrt{1 + \sec^4 x} dx$

7. $\int_{-1}^{2} \frac{|x|}{x} dx$ is

(A) -3 (B) 1 (C) 2 (D) 3 (E) nonexistent

8. The number of bacteria in a culture is growing at a rate of $3,000e^{2t/5}$ per unit of time t. At $t = 0$, the number of bacteria present was $7,500$. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

9. $\displaystyle\int_0^{\pi/4} \tan^2 x dx =$

(A) $\dfrac{\pi}{4} - 1$ (B) $1 - \dfrac{\pi}{4}$ (C) $\dfrac{1}{3}$ (D) $\sqrt{2} - 1$ (E) $\dfrac{\pi}{4} + 1$

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10. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 3
(B) 5
(C) 6
(D) 10
(E) 12

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. If $\dfrac{dy}{dx} = (1 + \ln x)y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1 + \ln x$

(C) $\ln x$

(D) $e^{2x + x\ln x - 2}$

(E) $e^{x \ln x}$

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x	2	5	7	8
$f(x)$	10	30	40	20

12. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

13. The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation?

(A) $\frac{dP}{dt} = \frac{1}{400}(1200 - P) + 200$

(B) $\frac{dP}{dt} = \frac{2}{5}(1200 - P)$

(C) $\frac{dP}{dt} = \frac{1}{500}P(1200 - P)$

(D) $\frac{dP}{dt} = \frac{1}{400}P(1200 - P)$

(E) $\frac{dP}{dt} = 400P(1200 - P)$

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14. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \leq t \leq 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)dt$

(B) $\int_{0}^{8} r(t)dt$

(C) $\int_{0}^{2.667} r(t)dt$

(D) $\int_{1.572}^{3.514} r'(t)dt$

(E) $\int_{0}^{2.667} r'(t)dt$

15. $\displaystyle\int \frac{dx}{(x-1)(x+3)} =$

(A) $\dfrac{1}{4}\ln\left|\dfrac{x-1}{x+3}\right| + C$

(B) $\dfrac{1}{4}\ln\left|\dfrac{x+3}{x-1}\right| + C$

(C) $\dfrac{1}{2}\ln|(x-1)(x+3)| + C$

(D) $\dfrac{1}{2}\ln\left|\dfrac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln|(x-1)(x+3)| + C$

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16. (EXTRA CREDIT, NO CALCULATOR).
Find the sum of the roots of the equation $\sqrt{x-1} + \sqrt{2x-1} = x$.

(A) 1 (B) 2 (C) 4 (D) 5 (E) 6