Name ____________________________________________
Date: April 30, 2019 Teacher: Asher Roberts

# AP Calculus BC Exam 4 Answer Sheet

Example: Ⓐ Ⓑ ● Ⓓ Ⓔ

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**CALCULUS BC**
**SECTION I, Part A**
**Time—20 minutes**
**Number of questions—10**

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

**Directions:** Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

**In this exam**:

(1) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$ ).

(for teacher use only)

| **Exam Score** | | |
|---|---|---|
| **Part** | **Number of Correct Answers** | **Number of Partially Correct Answers** |
| A | | |
| B | | |
| **Total:** | | |
| | **Overall Score:** | |

1. If $x = e^{2t}$ and $y = \sin(2t)$, then $\dfrac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\dfrac{e^{2t}}{\cos(2t)}$ (C) $\dfrac{\sin(2t)}{2e^{2t}}$ (D) $\dfrac{\cos(2t)}{2e^{2t}}$ (E) $\dfrac{\cos(2t)}{e^{2t}}$

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2. The sum of the infinite geometric series $\dfrac{3}{2} + \dfrac{9}{16} + \dfrac{27}{128} + \dfrac{81}{1,024} + \ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

3. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, tdt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

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4. Let $f$ be the function given by $f(x) = \ln(3-x)$. The third-degree Taylor polynomial for $f$ about $x = 2$ is

(A) $-(x-2) + \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

(B) $-(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

(C) $(x-2) + (x-2)^2 + (x-2)^3$

(D) $(x-2) + \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3}$

(E) $(x-2) - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3}$

5. For what values of $t$ does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only

(B) 1 only

(C) 0 and $\frac{2}{3}$ only

(D) 0, $\frac{2}{3}$, and 1

(E) No value

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6. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (B) $3\int_0^{\pi} \cos^2\theta d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta d\theta$ (E) $3\int_0^{\pi} \cos\theta d\theta$

7. The Taylor series for $\sin x$ about $x = 0$ is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$. If $f$ is a function such that $f'(x) = \sin(x^2)$, then the coefficient of $x^7$ in the Taylor series for $f(x)$ about $x = 0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) $0$ (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

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8. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

9. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ converges

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10. What is the slope of the line tangent to the polar curve $r = 2\theta$ at the point $\theta = \frac{\pi}{2}$?

(A) $-\frac{\pi}{2}$  (B) $-\frac{2}{\pi}$  (C) $0$  (D) $\frac{\pi}{2}$  (E) $2$

**CALCULUS BC**
**SECTION I, Part B**
**Time—15 minutes**
**Number of questions—5**

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

**Directions:** Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

**YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.**

**In this exam**:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$ ).

11. Which of the following sequences converge?

I. $\left\{\dfrac{5n}{2n-1}\right\}$

II. $\left\{\dfrac{e^n}{n}\right\}$

III. $\left\{\dfrac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

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12. For what integer $k$, $k > 1$, will both $\sum_{n=1}^{\infty} \dfrac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\dfrac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

13. The function $f$ has derivatives of all orders for all real numbers, and $f^{(4)}(x) = e^{\sin x}$. If the third-degree Taylor polynomial for $f$ about $x = 0$ is used to approximate $f$ on the interval $[0, 1]$, what is the Lagrange error bound for the maximum error on the interval $[0, 1]$?

(A) 0.019 (B) 0.097 (C) 0.113 (D) 0.399 (E) 0.417

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14. What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \leq x < -1$ (C) $-3 \leq x \leq -1$ (D) $-1 \leq x < 1$ (E) $-1 \leq x \leq 1$

15. The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

16. (EXTRA CREDIT, NO CALCULATOR).
Sketch the polar curve $r = 1 - 3\sin\theta$ and find the area enclosed by the inner loop.

**Definition.** We say that the infinite product $\prod_{n=1}^{\infty} a_n = a_1 a_2 \cdots$ converges if there is an integer $N \geq 2$ for which the limit

$$p = \lim_{k\to\infty} \prod_{n=N}^{k} a_n$$

exists and is non-zero. In this case, we set

$$\prod_{n=1}^{\infty} a_n = \lim_{k\to\infty} \prod_{n=1}^{k} a_n = a_1 \cdots a_{N-1} p.$$

17. (EXTRA CREDIT).
Let $\{a_n\}_{n=1}^{\infty}$ be a sequence. Use the infinite product

$$\prod_{n=2}^{\infty} \cos\left(\frac{\pi}{2^n}\right)$$

to deduce the value of

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots\cdot,$$

known as Vieta's formula (1579).
[Hint: Use the identities $\sin(2x) = 2\sin x \cos x$ and $\cos(2x) = 2\cos^2 x - 1$.]