Name ____

Date: December 15, 2020 Teacher: Asher Roberts

AP Calculus BC Exam 1 Answer Sheet

Example: (A) (B) ● (D) (E)

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score	
Part	**Number of Correct Answers**
A	
B	
Total:	
Overall Score:	

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

   (A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$

   (D) $f(x) = \log x$ (E) $f(x) = e^x$

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2. $\ln(x-2) < 0$ if and only if

   (A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$

   (D) $x > 2$ (E) $x > 3$

3. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

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4. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

5. $\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

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6. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

7. If $\dfrac{d}{dx}(f(x)) = g(x)$ and $\dfrac{d}{dx}(g(x)) = f(x^2)$, then $\dfrac{d^2}{dx^2}\left(f(x^3)\right) =$

(A) $f(x^6)$ (B) $g(x^3)$ (C) $3x^2 g(x^3)$

(D) $9x^4 f(x^6) + 6xg(x^3)$ (E) $f(x^6) + g(x^3)$

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8. If $f(x) = (x^2+1)^{2-3x}$, then $f'(1) =$

(A) $-\dfrac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\dfrac{3}{2}\ln(2)$ (D) $-\dfrac{1}{2}$ (E) $\dfrac{1}{8}$

9. If $y = \arctan(\cos x)$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{-\sin x}{1+\cos^2 x}$ (B) $-(\text{arcsec}(\cos x))^2 \sin x$ (C) $(\text{arcsec}(\cos x))^2$

(D) $\dfrac{1}{(\arccos x)^2+1}$ (E) $\dfrac{1}{1+\cos^2 x}$

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10. If $\lim_{x\to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x = a$.
(C) $f(x)$ is defined at $x = a$.
(D) $f(a) = L$
(E) None of the above

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$.
III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

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12. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

13. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

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14. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4+\frac{h}{32}$ (B) $2+\frac{h}{32}$ (C) $\frac{h}{32}$

(D) $4-\frac{h}{32}$ (E) $2-\frac{h}{32}$

x	0	1	2
$f(x)$	1	k	2

15. The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

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16. (EXTRA CREDIT). Use the precise definition of a limit to prove that if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$ both exist, then

$$\lim_{x \to a} [f(x) + g(x)] = L + M.$$