Name _____

Date: February 4, 2021 Teacher: Asher Roberts

AP Calculus BC Exam 2 Answer Sheet

Example: A B ● D E

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score	
Part	**Number of Correct Answers**
A	
B	
Total:	
Overall Score:	

1. A point moves on the x-axis in such a way that its velocity at time t ($t > 0$) is given by $v = \dfrac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

---

2. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\dfrac{e^4}{2} - e$ (B) $\dfrac{e^4}{2} - 1$ (C) $\dfrac{e^4}{2} - \dfrac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

3. A region in the plane is bounded by the graph of $y = \dfrac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \dfrac{1}{2}$; increases as m increases when $m > \dfrac{1}{2}$.
(E) increases as m increases when $m < \dfrac{1}{2}$; decreases as m increases when $m > \dfrac{1}{2}$.

---

4. $\displaystyle\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1.
(B) $-\dfrac{1}{2}$
(C) $\dfrac{1}{2}$
(D) 1
(E) none of the above

5. $\displaystyle\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\dfrac{1}{3}\ln e^{x^3} + C$ (B) $-\dfrac{e^{x^3}}{3} + C$ (C) $-\dfrac{1}{3e^{x^3}} + C$

(D) $\dfrac{1}{3}\ln e^{x^3} + C$ (E) $\dfrac{x^3}{3e^{x^3}} + C$

---

6. If n is a non-negative integer, then $\displaystyle\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n (B) n even, only (C) n odd, only

(D) nonzero n, only (E) all n

7. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

---

8. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

9. The point on the curve $2y = x^2$ nearest to $(4, 1)$ is

(A) $(0, 0)$ (B) $(2, 2)$ (C) $\left(\sqrt{2}, 1\right)$ (D) $\left(2\sqrt{2}, 4\right)$ (E) $(4, 8)$

---

10. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$
(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
(C) $-2 < x < 0$ or $x > 2$
(D) $x > \sqrt{2}$
(E) $-2 < x < 2$

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

---

12. Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $[0, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

13. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$
(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
(C) $2F(6) - 2F(2)$
(D) $F(6) - F(2)$
(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

---

14. If k is a positive integer, then $\lim_{x\to+\infty} \frac{x^k}{e^x}$ is

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

15. Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

(A) 125 (B) 100 (C) 88 (D) 50 (E) 12

---

16. (EXTRA CREDIT). If $f_{\text{ave}}[a,b]$ denotes the average value of f on the interval $[a,b]$ and $a < c < b$, show that

$$f_{\text{ave}}[a,b] = \frac{c-a}{b-a} f_{\text{ave}}[a,c] + \frac{b-c}{b-a} f_{\text{ave}}[c,b].$$