Name ____

Date: September 28, 2020 Teacher: Asher Roberts

Multivariable Calculus Exam 4

Vector Calculus
Time—55 minutes
Number of questions—10

A GRAPHING CALCULATOR MAY BE REQUIRED FOR SOME QUESTIONS ON THIS EXAM.

Directions: Solve each of the following problems, using the available space to show all relevant work. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 10 points. Use separate paper for scratch work, and do not turn in scratch work with your exam. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score			
Question	**Points**	**Question**	**Points**
1		6	
2		7	
3		8	
4		9	
5		10	10
Overall Score:			

1. Sketch the vector field $\mathbf{F}(x, y, z) = \mathbf{k}$ by drawing a diagram.

2. Evaluate the line integral $\int_C y^3 \, ds$ where C is the curve given by $x = t^3$, $y = t$, $0 \leq t \leq 2$.

3. Determine whether or not $\mathbf{F}(x, y) = (2x - 3y)\mathbf{i} + (-3x + 4y - 8)\mathbf{j}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

4. Evaluate the line integral $\oint_C (x - y)\, dx + (x + y)\, dy$, where C is the circle with center the origin and radius 2.

5. Find the curl and the divergence of the vector field $\mathbf{F}(x, y, z) = (x + yz)\mathbf{i} + (y + xz)\mathbf{j} + (z + xy)\mathbf{k}$.

6. Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes $z = -2$ and $z = 2$.

7. Evaluate the surface integral $\iint_S x^2 z^2 \, dS$, where S is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$.

8. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = x^2z^2\mathbf{i} + y^2z^2\mathbf{j} + xyz\mathbf{k}$, and S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$, oriented upward.

9. Calculate the flux of $\mathbf{F}(x, y, z) = x^2 \sin y\mathbf{i} + x \cos y\mathbf{j} - xz \sin y\mathbf{k}$ across S, where S is the "fat sphere" $x^8 + y^8 + z^8 = 8$.

10. Show that the divergence of a vector field can be defined as the transpose of the gradient of the vector field.