Multivariable Calculus Homework #6

Replace this text with your name

Due: Replace this text with a due date

Exercise (17.1.22)**.** Solve the initial-value problem

$$4y'' - 20y' + 25y = 0 \qquad y(0) = 2 \qquad y'(0) = -3.$$

Solution: Replace this text with your solution. □

Exercise (17.1.32)**.** Solve the boundary-value problem

$$y'' + 4y' + 20y = 0 \qquad y(0) = 1 \qquad y(\pi) = e^{-2\pi},$$

if possible.

Solution: Replace this text with your solution. □

Exercise (17.1.33)**.** Let L be a nonzero real number.

(a) Show that the boundary-value problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(L) = 0$ has only the trivial solution $y = 0$ for the cases $\lambda = 0$ and $\lambda < 0$.

(b) For the case $\lambda > 0$, find the values of λ for which this problem has a nontrivial solution and give the corresponding solution.

Solution: Replace this text with your solution. □

Exercise (17.1.34)**.** If a, b, and c are all positive constants and $y(x)$ is a solution of the differential equation $ay'' + by' + cy = 0$, show that $\lim_{x\to\infty} y(x) = 0$.

Solution: Replace this text with your solution. □

Exercise (17.2.10)**.** Solve the initial-value problem

$$y'' + y' - 2y = x + \sin 2x \qquad y(0) = 1 \qquad y'(0) = 0.$$

Solution: Replace this text with your solution. □

Exercise (17.2.17)**.** Write a trial solution for

$$y'' + 2y' + 10y = x^2 e^{-x} \cos 3x$$

for the method of undetermined coefficients. Do not determine the coefficients.

Solution: Replace this text with your solution. □

Exercise (17.2.21)**.** Solve the differential equation

$$y'' - 2y' + y = e^{2x}$$

using (a) undetermined coefficients and (b) variation of parameters.

Solution: Replace this text with your solution. □

Exercise (17.2.28)**.** Solve the differential equation

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}$$

using the method of variation of parameters.

Solution: Replace this text with your solution. □

Exercise (17.3.9)**.** Suppose a spring has mass m and spring constant k and let $\omega = \sqrt{k/m}$. Suppose that the damping constant is so small that the damping force is negligible. If an external force $F(t) = F_0 \cos \omega_0 t$ is applied, where $\omega_0 \neq \omega$, use the method of undetermined coefficients to show that the motion of the mass is described by

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0}{m(\omega^2 - \omega_0^2)} \cos \omega_0 t.$$

Solution: Replace this text with your solution. □

Exercise (17.3.10)**.** As in Exercise 17.3.9, consider a spring with mass m, spring constant k, and damping constant $c = 0$, and let $\omega = \sqrt{k/m}$. If an external force $F(t) = F_0 \cos \omega t$ is applied (the applied frequency equals the natural frequency), use the method of undetermined coefficients to show that the motion of the mass is given by

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0}{2m\omega} t \sin \omega t.$$

Solution: Replace this text with your solution. □

Exercise (17.3.12)**.** Consider a spring subject to a frictional or damping force.

(a) In the critically damped case, the motion is given by $x = c_1 e^{rt} + c_2 t e^{rt}$. Show that the graph of x crosses the t-axis whenever c_1 and c_2 have opposite signs.

(b) In the overdamped case, the motion is given $x = c_1 e^{r_1 t} + c_2 e^{r_2 t}$, where $r_1 > r_2$. Determine a condition on the relative magnitudes of c_1 and c_2 under which the graph of x crosses the t-axis at a positive value of t.

Solution: Replace this text with your solution. □

Exercise (17.3.13)**.** A series circuit consists of a resistor with $R = 20\ \Omega$, an inductor with $L = 1$ H, a capacitor with $C = 0.002$ F, and a 12-V battery. If the initial charge and current are both 0, find the charge and current at time t.

Solution: Replace this text with your solution. □

Exercise (17.3.15)**.** The battery in Exercise 17.3.13 is replaced by a generator producing a voltage of $E(t) = 12 \sin 10t$. Find the charge at time t.

Solution: Replace this text with your solution. □

Exercise (17.4.8)**.** Use power series to solve the differential equation

$$y'' = xy.$$

Solution: Replace this text with your solution. □

Exercise (17.4.11)**.** Use power series to solve the differential equation

$$y'' + x^2y' + xy = 0 \qquad y(0) = 0 \qquad y'(0) = 1.$$

Solution: Replace this text with your solution. □

Exercise (17.4.12)**.** The solution of the initial-value problem

$$x^2y'' + xy' + x^2y = 0 \qquad y(0) = 1 \qquad y'(0) = 0$$

is called a Bessel function of order 0.

(a) Solve the initial-value problem to find a power series expansion for the Bessel function.

(b) Graph several Taylor polynomials until you reach one that looks like a good approximation to the Bessel function on the interval $[-5, 5]$.

Solution: Replace this text with your solution. □