Name _____

Date: April 20, 2021 Teacher: Asher Roberts

AP Calculus BC Exam 4 Answer Sheet

Example: A B ● D E

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score	
Part	**Number of Correct Answers**
A	
B	
Total:	
Overall Score:	

1. If $x = t^2 - 1$ and $y = 2e^t$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{e^t}{t}$ (B) $\dfrac{2e^t}{t}$ (C) $\dfrac{e^{|t|}}{t^2}$ (D) $\dfrac{4e^t}{2t-1}$ (E) e^t

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2. A series expansion of $\dfrac{\sin t}{t}$ is

(A) $1 - \dfrac{t^2}{3!} + \dfrac{t^4}{5!} - \dfrac{t^6}{7!} + \cdots$

(B) $\dfrac{1}{t} - \dfrac{t}{2!} + \dfrac{t^3}{4!} - \dfrac{t^5}{6!} + \cdots$

(C) $1 + \dfrac{t^2}{3!} + \dfrac{t^4}{5!} + \dfrac{t^6}{7!} + \cdots$

(D) $\dfrac{1}{t} + \dfrac{t}{2!} + \dfrac{t^3}{4!} + \dfrac{t^5}{6!} + \cdots$

(E) $t - \dfrac{t^3}{3!} + \dfrac{t^5}{5!} - \dfrac{t^7}{7!} + \cdots$

3. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

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4. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln \frac{5}{2}\right)$

(D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln \frac{3}{2}\right)$

5. The area of the region enclosed by the polar curve $r = 1 - \cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

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6. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges.

7. A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x \sin x + 3x^2$ (B) $-\cos(x^2) + 1$ (C) $-x^2 \cos x + x^2$

(D) $x^2 e^x - x^3 - x^2$ (E) $e^{x^2} - x^2 - 1$

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8. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$ (B) $p \geq 1$ (C) $p > 1$ (D) $p \geq 2$ (E) $p > 2$

9. What is the coefficient of x^2 in the Taylor series for $\dfrac{1}{(1+x)^2}$ about $x = 0$?

(A) $\dfrac{1}{6}$ (B) $\dfrac{1}{3}$ (C) 1 (D) 3 (E) 6

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10. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\displaystyle\lim_{n\to\infty} \frac{e}{n!} < 1$

(B) $\displaystyle\lim_{n\to\infty} \frac{n!}{e} < 1$

(C) $\displaystyle\lim_{n\to\infty} \frac{n+1}{e} < 1$

(D) $\displaystyle\lim_{n\to\infty} \frac{e}{n+1} < 1$

(E) $\displaystyle\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$

(D) $(e^{-t}, \cos t)$ (E) $(e^{-t}, -\cos t)$

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12. The Taylor series for $\ln x$, centered at $x = 1$, is $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

13. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) -30 (B) -15 (C) -5 (D) $-\frac{5}{6}$ (E) $-\frac{1}{6}$

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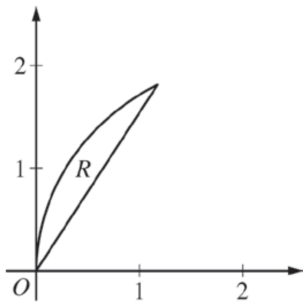
14. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim_{n\to\infty} a_n = k$ (B) $\int_1^n f(x)\,dx = k$ (C) $\int_1^\infty f(x)\,dx$ diverges

(D) $\int_1^\infty f(x)\,dx$ converges (E) $\int_1^\infty f(x)\,dx = k$



15. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819