Name _____

Date: December 9, 2022 Teacher: Asher Roberts

AP Calculus BC Exam 1 Answer Sheet

Example: (A) (B) (●) (D) (E)

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score		
Part	**Number of Correct Answers**	**Number of Partially Correct Answers**
A		
B		
Total:		
	Overall Score:	

1. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

(A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125

(D) 225 (E) 5

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2. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \dfrac{x+1}{x}$ (B) $y = -x^5 + 3x$ (C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$ (E) $y = (x^2+1)^2 - 1$

3. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

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4. If $y = \cos^2(3x)$, then $\frac{dy}{dx} =$

(A) $-6\sin(3x)\cos(3x)$ (B) $-2\cos(3x)$ (C) $2\cos(3x)$

(D) $6\cos(3x)$ (E) $2\sin(3x)\cos(3x)$

5. If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$

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6. If $f(x) = \arccos(x^2)$, then $f'(x) =$

(A) $\dfrac{1}{\sqrt{1-x^4}}$

(B) $\dfrac{-2x}{\sqrt{1-x^4}}$

(C) $\dfrac{2x}{\sqrt{1-x^4}}$

(D) $\dfrac{-4x^3}{\sqrt{1-x^4}}$

(E) $\dfrac{4x^3}{\sqrt{1-x^4}}$

7. Let f and g be differentiable functions such that

$$\begin{array}{lll} f(1) = 2, & f'(1) = 3, & f'(2) = -4, \\ g(1) = 2, & g'(1) = -3, & g'(2) = 5. \end{array}$$

If $h(x) = f(g(x))$, then $h'(1) =$

(A) -9 (B) -4 (C) 0 (D) 12 (E) 15

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8. Which of the following functions are continuous for all real numbers x?

I. $y = x^{\frac{2}{3}}$
II. $y = e^x$
III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

9. If $f(x) = \dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

(A) 2 (B) $\dfrac{1}{2}$ (C) $1 + \dfrac{\pi}{2}$ (D) $\dfrac{\pi}{2} - 1$ (E) $1 - \dfrac{\pi}{2}$

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10. If f is a function such that $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} = 0$, which of the following must be true?

(A) The limit of $f(x)$ as x approaches 2 does not exist.
(B) f is not defined at $x = 2$.
(C) The derivative of f at $x = 2$ is 0.
(D) f is continuous at $x = 0$.
(E) $f(2) = 0$.

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

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12. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

13. If $a \neq 0$, then $\lim_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

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14. Population y grows according to the equation $\dfrac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

15. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle when the circumference of the circle is 20π meters?

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

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16. (EXTRA CREDIT). Assume

$$\lim_{x \to a} f(x) = L.$$

If $f(x) \geq k$ where k is a constant, prove that $L \geq k$.