Name _____

Date: January 25, 2023 Teacher: Asher Roberts

AP Calculus BC Exam 2 Answer Sheet

Example: (A) (B) (●) (D) (E)

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score		
Part	**Number of Correct Answers**	**Number of Partially Correct Answers**
A		
B		
Total:		
	Overall Score:	

1. $\int (x^3 - 3x)\,dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

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2. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

3. The *derivative* of $f(x) = \dfrac{x^4}{3} - \dfrac{x^5}{5}$ attains its maximum value at $x =$

(A) -1 (B) 0 (C) 1 (D) $\dfrac{4}{3}$ (E) $\dfrac{5}{3}$

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4. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{x/2}$ is

(A) $\dfrac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

5. $\displaystyle\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\dfrac{e^3}{2}$ (B) $\dfrac{e^3-1}{2}$ (C) $\dfrac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

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6. $\displaystyle\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\dfrac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

7. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

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8. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

9. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

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10. $\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right] =$

(A) $\frac{1}{2}\int_0^1 \frac{1}{\sqrt{x}}\,dx$ (B) $\int_0^1 \sqrt{x}\,dx$ (C) $\int_0^1 x\,dx$

(D) $\int_1^2 x\,dx$ (E) $2\int_1^2 x\sqrt{x}\,dx$

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

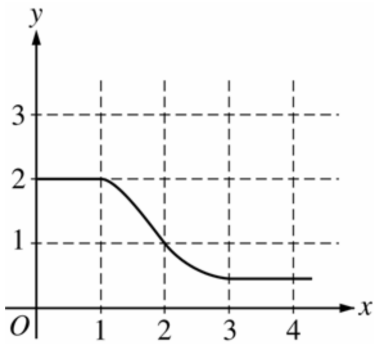
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33



12. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

13. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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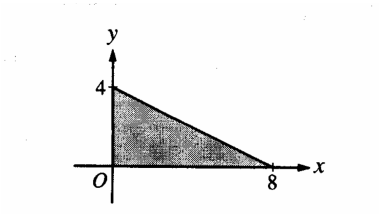
14. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \leq x \leq 3$?

I. The average rate of change of f is $\dfrac{5}{2}$.

II. The average value of f is $\dfrac{9}{2}$.

III. The average value of f' is $\dfrac{5}{2}$.

(A) None (B) I only (C) III only (D) I and III only (E) II and III only



15. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

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16. (EXTRA CREDIT). Prove

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$