Name ____
Date: March 24, 2023 Teacher: Asher Roberts

AP Calculus BC Exam 3 Answer Sheet

Example: (A) (B) ● (D) (E)

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score		
Part	**Number of Correct Answers**	**Number of Partially Correct Answers**
A		
B		
Total:		
	Overall Score:	

1. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\displaystyle\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\displaystyle\int_0^{1/2} \sin^2 y\,dy$ (B) $\displaystyle 2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$ (C) $\displaystyle 2\int_0^{\pi/4} \sin^2 y\,dy$

(D) $\displaystyle\int_0^{\pi/4} \sin^2 y\,dy$ (E) $\displaystyle 2\int_0^{\pi/6} \sin^2 y\,dy$

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2. $\displaystyle\int_0^{1} \frac{x+1}{x^2+2x-3}\,dx$ is

(A) $-\ln\sqrt{3}$ (B) $-\dfrac{\ln\sqrt{3}}{2}$ (C) $\dfrac{1-\ln\sqrt{3}}{2}$ (D) $\ln\sqrt{3}$ (E) divergent

3. $\int \tan(2x)\, dx =$

(A) $-2\ln|\cos(2x)| + C$

(B) $-\frac{1}{2}\ln|\cos(2x)| + C$

(C) $\frac{1}{2}\ln|\cos(2x)| + C$

(D) $2\ln|\cos(2x)| + C$

(E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

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4. What is the length of the arc of $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$?

(A) $\frac{8}{3}$

(B) 4

(C) $\frac{14}{3}$

(D) $\frac{16}{3}$

(E) 7

5. $\displaystyle\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$

(A) $-\dfrac{33}{20}$ (B) $-\dfrac{9}{20}$ (C) $\ln\left(\dfrac{5}{2}\right)$ (D) $\ln\left(\dfrac{8}{5}\right)$ (E) $\ln\left(\dfrac{2}{5}\right)$

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6. If $\dfrac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\dfrac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\dfrac{x^3}{3} + 1$

7. $\int x \sec^2 x \, dx =$

(A) $x \tan x + C$

(B) $\dfrac{x^2}{2} \tan x + C$

(C) $\sec^2 x + 2 \sec^2 x \tan x + C$

(D) $x \tan x - \ln |\cos x| + C$

(E) $x \tan x + \ln |\cos x| + C$

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8. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}} \, dx$ converges?

(A) $p < -1$

(B) $p > 0$

(C) $p > \dfrac{1}{2}$

(D) $p > 1$

(E) There are no values of p for which this integral converges.

9. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

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10. $\int \frac{x^3 - x - 1}{(x+1)^2}\, dx =$

(A) $(x-2) + \frac{2x+1}{(x+1)^2} + C$

(B) $x^2 - 2x + \frac{1}{2}\ln(x^2 + 2x + 1) + C$

(C) $\frac{1}{2}x^2 - 2x + \ln|x+1|^2 - \frac{1}{x+1} + C$

(D) $\frac{1}{2}(x-2)^2 + 2\ln|x+1| + \frac{1}{x+1} + C$

(E) none of the above

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

11. A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

    (A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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12. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

    (A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

13. If f is a function such that $f'(x) = -f(x)$, then $\int xf(x)\,dx =$

(A) $f(x)(x+1) + C$

(B) $-f(x)(x+1) + C$

(C) $\dfrac{x^2}{2}f(x) + C$

(D) $-\dfrac{x^2}{2}f(x) + C$

(E) $-\dfrac{x^2}{2}f(x)\left(1 + \dfrac{x}{3}\right) + C$

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14. $\displaystyle\int \frac{dx}{x^2 + 6x + 10} =$

(A) $\cot^{-1}(x+3) + C$

(B) $\sin^{-1}(x+3) + C$

(C) $\sec^{-1}(x+3) + C$

(D) $\tan^{-1}(x+3) + C$

(E) $\cos^{-1}(x+3) + C$

15. The number of students in a school who have heard a rumor at time t hours is modeled by the function P, the solution to a logistic differential equation. At noon, 50 of the school's 500 students have heard the rumor. Also at noon, P is increasing at a rate of 20 students per hour. Which of the following could be the logistic differential equation?

(A) $\dfrac{dP}{dt} = \dfrac{1}{1125}P(500 - P)$

(B) $\dfrac{dP}{dt} = \dfrac{1}{480}P(500 - P)$

(C) $\dfrac{dP}{dt} = \dfrac{1}{192}P(500 - P)$

(D) $\dfrac{dP}{dt} = \dfrac{2}{45}P(500 - P)$

(E) $\dfrac{dP}{dt} = \dfrac{5}{48}P(500 - P)$

16. (EXTRA CREDIT). In the case where f is positive and has a continuous derivative, we define the surface area of the surface obtained by rotating the curve $y = f(x)$, $a \leq x \leq b$, about the x-axis as

$$\begin{aligned} S &= \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi f(x_i^*) \sqrt{1 + [f'(x_i^*)]^2} \Delta x \\ &= \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx. \end{aligned}$$

Using this definition, determine the surface area of a solid *torus* (the donut-shaped solid shown in the figure) with radii r and R.