Name ______________________________

Date: April 28, 2023 Teacher: Asher Roberts

AP Calculus BC Exam 4 Answer Sheet

Example: (A) (B) ● (D) (E)

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)

CALCULUS BC
SECTION I, Part A
Time—20 minutes
Number of questions—10

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. Two credits will be given for each correct answer, and one credit may be given for incorrect answers where there is correct work written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

(for teacher use only)

Exam Score		
Part	**Number of Correct Answers**	**Number of Partially Correct Answers**
A		
B		
Total:		
	Overall Score:	

1. For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) No values of x (B) $x < -1$ (C) $x \geq -1$ (D) $x > -1$ (E) All values of x

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2. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

(A) $0 < x < 2$ (B) $0 \leq x \leq 2$ (C) $-2 < x \leq 0$

(D) $-2 \leq x < 0$ (E) $-2 \leq x \leq 0$

3. Which of the following series converges for all real numbers x?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ (B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ (D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ (E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

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4. For time $t > 0$, the position of a particle moving in the xy-plane is given by the parametric equations $x = 4t + t^2$ and $y = \dfrac{1}{3t+1}$. What is the acceleration vector of the particle at time $t = 1$?

(A) $\left(2, \frac{1}{32}\right)$

(B) $\left(2, \frac{9}{32}\right)$

(C) $\left(5, \frac{1}{4}\right)$

(D) $\left(6, -\frac{3}{16}\right)$

(E) $\left(6, -\frac{1}{16}\right)$

5. $\displaystyle\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^{i} =$

(A) $\dfrac{3}{2}-\left(\dfrac{1}{3}\right)^{n}$ (B) $\dfrac{3}{2}\left[1-\left(\dfrac{1}{3}\right)^{n}\right]$ (C) $\dfrac{3}{2}\left(\dfrac{1}{3}\right)^{n}$

(D) $\dfrac{2}{3}\left(\dfrac{1}{3}\right)^{n}$ (E) $\dfrac{2}{3}\left(\dfrac{1}{3}\right)^{n+1}$

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6. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is

(A) $y = 2x$ (B) $y = 8x$ (C) $y = 2x - 1$

(D) $y = 4x - 5$ (E) $y = 8x + 13$

7. Which of the following series converge?

I. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^n$

III. $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III

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8. Which of the following gives the length of the path described by the parametric equations $x = \sin(t^3)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{\sin^2(t^3) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2(t^3) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4\cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2\cos(t^3) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2(3t^2) + e^{10t}}\, dt$

9. The power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at $x=5$. Which of the following must be true?

(A) The series diverges at $x=0$.
(B) The series diverges at $x=1$.
(C) The series converges at $x=1$.
(D) The series converges at $x=2$.
(E) The series converges at $x=6$.

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10. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

CALCULUS BC
SECTION I, Part B
Time—15 minutes
Number of questions—5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. One credit will be given for each correct answer. Do not spend too much time on any one problem.

YOU MAY NOT RETURN TO PROBLEMS 1-10 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

11. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909
(B) 3.062
(C) 6.884
(D) 9.016
(E) 47.393

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12. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.
(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.
(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.
(D) $\sum_{n=1}^{\infty} b_n$ converges.
(E) $\sum_{n=1}^{\infty} b_n$ diverges.

13. Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \leq |a_n|$ for all $n \geq 2$

III. $\lim_{n \to \infty} a_n = 0$

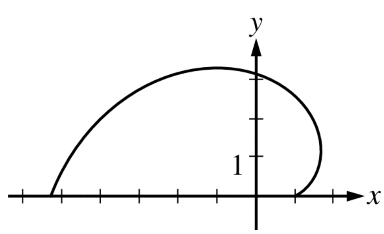
(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III



14. The graph above shows the polar curve $r = 2\theta + \cos\theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x-axis?

(A) 3.069 (B) 4.935 (C) 9.870 (D) 17.456 (E) 34.912

15. Let f be a positive, continuous, decreasing function. If $\int_1^{\infty} f(x)\,dx = 5$, which of the following statements about the series $\sum_{n=1}^{\infty} f(n)$ must be true?

(A) $\sum_{n=1}^{\infty} f(n) = 0$

(B) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) < 5$.

(C) $\sum_{n=1}^{\infty} f(n) = 5$

(D) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) > 5$.

(E) $\sum_{n=1}^{\infty} f(n)$ diverges

Definition. We say that the infinite product $\prod_{n=1}^{\infty} a_n = a_1 a_2 \cdots$ converges if there is an integer $N \geq 2$ for which the limit

$$p = \lim_{k \to \infty} \prod_{n=N}^{k} a_n$$

exists and is non-zero. In this case, we set

$$\prod_{n=1}^{\infty} a_n = \lim_{k \to \infty} \prod_{n=1}^{k} a_n = a_1 \cdots a_{N-1} p.$$

16. (EXTRA CREDIT).
Let $s > 1$ and p_k be the kth prime. Rewrite the product

$$\prod_{k=1}^{\infty} \frac{1}{1 - \frac{1}{p_k^s}}$$

in the form

$$\sum_{n=1}^{\infty} a_n(s),$$

for a sequence $\{a_n\}_{n=1}^{\infty}$ that depends on s.
[Hint: Use the fundamental theorem of arithmetic, which states that every integer greater than 1 can be represented uniquely as a product of prime factors.]