Calculus I Test 3

Derivatives Number of questions—10

Directions: Solve each of the following problems, using the available space to show all relevant work. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 10 points. You must work alone, but you may use a graphing calculator as a supplement to your own work if you indicate the steps used. You may not use computational intelligence or AI.

In this exam:

- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.
- (2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

| Exam Score | | | | | |
|-------------------|--------|----------|--------|--|--|
| Question | Points | Question | Points | | |
| 1 | | 6 | | | |
| 2 | | 7 | | | |
| 3 | | 8 | | | |
| 4 | | 9 | | | |
| 5 | | 10 | | | |
| Overall Score: | | | | | |

(for teacher use only)

1. Find f'(x) if $f(x) = (3x - 9)^{-2}(2x - 6)^4$.

2. Find
$$\frac{d}{dx} (\log_7(\log_4 x)).$$

3. Find
$$\frac{d}{dx}(\ln 5^{3x})$$
.

4. If
$$xy^2 + 2xy = 8$$
, what is $\frac{dy}{dx}$ at the point $(1, 2)$?

5. Find
$$\frac{d^2y}{dx^2}$$
 for $x^4 + y^4 = 100$. Simplify where possible.

6. What is an equation for a tangent to the graph of $y = \arctan \frac{x}{4}$ at x = 0?

7. Use the table below to find $(f^{-1})'(3)$.

| x | f(x) | f'(x) | $f^{-1}(x)$ |
|---|------|-------|-------------|
| 3 | -2 | 5 | 4 |
| 4 | 3 | -2 | -7 |

8. If a snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$, find the rate at which the diameter decreases when the diameter is 10 cm.

9. Let $f(x) = 2x^3 + x^2 - 4x$. Where does f have critical points?

10. Find the absolute maximum and absolute minimum values of $f(x) = 2\cos x + \sin 2x$ on the interval $[0, \pi/2]$.