Calculus III Homework #2

Replace this text with your name

Due: Replace this text with a due date

Exercise (13.1.20)**.** Find a vector equation and parametric equations for the line segment that joins $P(a, b, c)$ to $Q(u, v, w)$.

Solution: Replace this text with your solution. $\square$

Exercise (13.1.32)**.** At what points does the helix $\mathbf{r}(t) = \langle \sin t, \cos t, t\rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

Solution: Replace this text with your solution. $\square$

Exercise (13.1.37)**.** Use a computer to graph the curve $\mathbf{r}(t) = \langle \cos 2t, \cos 3t, \cos 4t\rangle$. Make sure you choose a parameter domain and viewpoints that reveal the true nature of the curve.

Solution: Replace this text with your solution. $\square$

Exercise (13.1.45)**.** Find a vector function that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

Solution: Replace this text with your solution. $\square$

Exercise (13.1.49)**.** If two objects travel through space along two different curves, it's often important to know whether they will collide. (Will a missile hit its moving target? Will two aircraft collide?) The curves might intersect, but we need to know whether the objects are in the same position *at the same time*. Suppose the trajectories of two particles are given by the vector functions

$$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2\rangle \qquad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6\rangle$$

for $t \geq 0$. Do the particles collide?

Solution: Replace this text with your solution. $\square$

Exercise (13.2.22)**.** If $\mathbf{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t}\rangle$, find $\mathbf{T}(0)$, $\mathbf{r}''(0)$, and $\mathbf{r}'(t)\cdot\mathbf{r}''(t)$.

Solution: Replace this text with your solution. □

Exercise (13.2.25)**.** Find parametric equations for the tangent line to the curve with parametric equations $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$ at $(1, 0, 1)$.

Solution: Replace this text with your solution. □

Exercise (13.2.49)**.** Find $f'(2)$, where $f(t) = \mathbf{u}(t)\cdot\mathbf{v}(t)$, $\mathbf{u}(2) = \langle 1, 2, -1\rangle$, $\mathbf{u}'(2) = \langle 3, 0, 4\rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3\rangle$.

Solution: Replace this text with your solution. □

Exercise (13.2.54)**.** Find an expression for $\dfrac{d}{dt}[\mathbf{u}(t)\cdot(\mathbf{v}(t)\times\mathbf{w}(t))]$.

Solution: Replace this text with your solution. □

Exercise (13.2.57)**.** If $\mathbf{u}(t) = \mathbf{r}(t)\cdot[\mathbf{r}'(t)\times\mathbf{r}''(t)]$, show that

$$\mathbf{u}'(t) = \mathbf{r}(t)\cdot[\mathbf{r}'(t)\times\mathbf{r}'''(t)].$$

Solution: Replace this text with your solution. □

Exercise (13.3.56)**.** Show that the osculating plane at every point on the curve $\mathbf{r}(t) = \langle t+2, 1-t, \frac{1}{2}t^2\rangle$ is the same plane. What can you conclude about the curve?

Solution: Replace this text with your solution. $\square$

Exercise (13.3.59)**.** Show that the curvature κ is related to the tangent and normal vectors by the equation

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}.$$

Solution: Replace this text with your solution. $\square$

Exercise (13.3.61)**.** (a) Show that $d\mathbf{B}/ds$ is perpendicular to $\mathbf{B}$.

(b) Show that $d\mathbf{B}/ds$ is perpendicular to $\mathbf{T}$.

(c) Deduce from parts (a) and (b) that $d\mathbf{B}/ds = -\tau(s)\mathbf{N}$ for some number $\tau(s)$ called the torsion of the curve. (The torsion measures the degree of twisting of a curve.)

(d) Show that for a plane curve the torsion is $\tau(s) = 0$.

Solution: Replace this text with your solution. $\square$

Exercise (13.3.62)**.** The following formulas, called the Frenet-Serret formulas, are of fundamental importance in differential geometry:

1. $d\mathbf{T}/ds = \kappa\mathbf{N}$

2. $d\mathbf{N}/ds = -\kappa\mathbf{T} + \tau\mathbf{B}$

3. $d\mathbf{B}/ds = -\tau\mathbf{N}$

Use the fact that $\mathbf{N} = \mathbf{B} \times \mathbf{T}$ to deduce Formula 2 from Formula 1 and 3.

Solution: Replace this text with your solution. $\square$

Exercise (13.3.67)**.** The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 Å $= 10^{-8}$ cm). Each helix rises about 34 Å during each complete turn, and there are about 2.9×10^8 complete turns. Estimate the length of each helix.

Solution: Replace this text with your solution. $\square$