Calculus III Homework #4

Replace this text with your name

Due: Replace this text with a due date

Exercise (15.1.6)**.** A 20-ft-by-30-ft swimming pool is filled with water. The depth is measured at 5-ft intervals, starting at one corner of the pool, and the values are recorded in the table. Estimate the volume of water in the pool.

	0	5	10	15	20	25	30
0	2	3	4	6	7	8	8
5	2	3	4	7	8	10	8
10	2	4	6	8	10	12	10
15	2	3	4	5	6	8	7
20	2	2	2	2	3	4	4

Solution: Replace this text with your solution. □

Exercise (15.1.11)**.** Evaluate the double integral

$$\iint_R (4-2y)\,dy, \quad R=[0,1]\times[0,1]$$

by first identifying it as the volume of a solid.

Solution: Replace this text with your solution. □

Exercise (15.1.49)**.** Find the volume of the solid enclosed by the paraboloid $z=2+x^2+(y-2)^2$ and the planes $z=1$, $x=1$, $x=-1$, $y=0$, and $y=4$.

Solution: Replace this text with your solution. □

Exercise (15.1.55)**.** Use symmetry to evaluate the double integral

$$\iint_R \frac{xy}{1+x^4}\,dA, \quad R=\{(x,y)\mid -1\le x\le 1, 0\le y\le 1\}.$$

Solution: Replace this text with your solution. □

Exercise (15.1.57)**.** Use a computer algebra system to compute the iterated integrals

$$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx \qquad \text{and} \qquad \int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy.$$

Do the answers contradict Fubini's Theorem? Explain what is happening.

Solution: Replace this text with your solution. $\square$

Exercise (15.2.45)**.** Find the volume of the solid under the plane $z = 3$, above the plane $z = y$, and between the parabolic cylinders $y = x^2$ and $y = 1 - x^2$, by subtracting two volumes.

Solution: Replace this text with your solution. □

Exercise (15.2.66)**.** Evaluate the integral

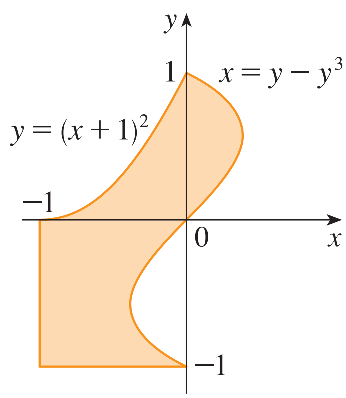
$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy$$

by reversing the order of integration.

Solution: Replace this text with your solution. □

Exercise (15.2.68)**.** Express D as a union of regions of type I or type II and evaluate the integral

$$\iint_D y\,dA.$$



Solution: Replace this text with your solution. □

Exercise (15.2.73)**.** Prove that if $m \le f(x, y) \le M$ for all x, y in D, then

$$mA(D) \le \iint_D f(x, y)\,dA \le MA(D).$$

Solution: Replace this text with your solution. □

Exercise (15.2.79)**.** Use geometry or symmetry, or both, to evaluate the double integral

$$\iint_D \left(ax^3 + by^3 + \sqrt{a^2 - x^2}\right) dA, \quad D = [-a, a] \times [-b, b].$$

Solution: Replace this text with your solution. □

Exercise (15.3.49)**.** Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\,dy\,dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,dy\,dx$$

into one double integral. Then evaluate the double integral.

Solution: Replace this text with your solution. □

Exercise (15.3.50)**.** (a) We define the improper integral (over the entire plane $\mathbb{R}^2$)

$$\begin{aligned} I &= \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dA \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dy\,dx \\ &= \lim_{a\to\infty}\iint_{D_a} e^{-(x^2+y^2)}\,dA, \end{aligned}$$

where D_a is the disk with radius a and center the origin.
Show that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dA = \pi.$$

(b) An equivalent definition of the improper integral in part (a) is

$$\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dA = \lim_{a\to\infty}\iint_{S_a} e^{-(x^2+y^2)}\,dA,$$

where S_a is the square with vertices $(\pm a, \pm a)$. Use this to show that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx \int_{-\infty}^{\infty} e^{-y^2}\,dy = \pi.$$

(c) Deduce that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}.$$

(d) By making the change of variable $t = \sqrt{2}x$, show that

$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}.$$

(This is a fundamental result for probability and statistics.)

Solution: Replace this text with your solution. □

Exercise (15.6.28). (a) In the Midpoint Rule for triple integrals we use a triple Riemann sum to approximate a triple integral over a box B, where $f(x,y,z)$ is evaluated at the center $(\bar{x}_i, \bar{y}_j, \bar{z}_k)$ of the box B_{ijk}. Use the Midpoint Rule to estimate $\iiint_B \sqrt{x^2+y^2+z^2}\,dV$, where B is the cube defined by $0 \le x \le 4$, $0 \le y \le 4$, $0 \le z \le 4$. Divide B into eight cubes of equal size.

(b) Use a computer algebra system to approximate the integral in part (a) correct to the nearest integer. Compare with the answer to part (a).

Solution: Replace this text with your solution. ☐

Exercise (15.6.40). Write five other iterated integrals that are equal to the iterated integral

$$\int_0^1 \int_y^1 \int_0^z f(x,y,z)\,dx\,dz\,dy.$$

Solution: Replace this text with your solution. ☐

Exercise (15.6.42). Evaluate the triple integral $\iiint_B (z^3+\sin y+3)\,dV$, where B is the unit ball $x^2+y^2+z^2 \le 1$, using only geometric interpretation and symmetry.

Solution: Replace this text with your solution. ☐

Exercise (15.6.52). Set up, but do not evaluate, integral expressions for (a) the mass, (b) the center of mass, and (c) the moment of inertia about the z-axis for the hemisphere $x^2+y^2+z^2 \le 1$, $z \ge 0$; $\rho(x,y,z) = \sqrt{x^2+y^2+z^2}$.

Solution: Replace this text with your solution. ☐

Exercise (15.6.57). The average value of a function $f(x,y,z)$ over a solid region E is defined to be

$$f_{\text{ave}} = \frac{1}{V(E)} \iiint_E f(x,y,z)\,dV$$

where $V(E)$ is the volume of E. For instance, if ρ is a density function then ρ_{ave} is the average density of E.
Find the average value of the function $f(x,y,z) = xyz$ over the cube with side length L that lies in the first octant with one vertex at the origin and edges parallel to the coordinate axes.

Solution: Replace this text with your solution. ☐

Exercise (15.7.13)**.** A cylindrical shell is 20 cm long, with inner radius 6 cm and outer radius 7 cm. Write inequalities that describe the shell in an appropriate coordinate system. Explain how you have positioned the coordinate system with respect to the shell.

Solution: Replace this text with your solution. □

Exercise (15.7.24)**.** Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

Solution: Replace this text with your solution. □

Exercise (15.7.32)**.** Evaluate the integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2 + y^2}\, dz\, dy\, dx$$

by changing to cylindrical coordinates.

Solution: Replace this text with your solution. □

Exercise (15.7.33)**.** When studying the formation of mountain ranges, geologists estimate the amount of work required to lift a mountain from sea level. Consider a mountain that is essentially in the shape of a right circular cone. Suppose that the weight density of the material in the vicinity of a point P is $g(P)$ and the height is $h(P)$.

(a) Find a definite integral that represents the total work done in forming the mountain.

(b) Assume that Mount Fuji in Japan is in the shape of a right circular cone with radius 62,000 ft, height 12,400 ft, and density a constant 200 lb/ft^3. How much work was done in forming Mount Fuji if the land was initially at sea level?

Solution: Replace this text with your solution. □

Exercise (15.8.38)**.** Find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\pi/6$.

Solution: Replace this text with your solution. $\square$

Exercise (15.8.43)**.** Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\,dz\,dy\,dx$$

by changing to spherical coordinates.

Solution: Replace this text with your solution. $\square$

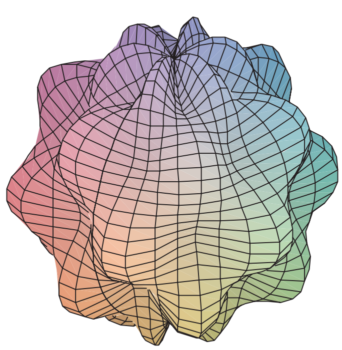
Exercise (15.8.46)**.** A model for the density δ of the earth's atmosphere near its surface is

$$\delta = 619.09 - 0.000097\rho$$

where ρ (the distance from the center of the earth) is measured in meters and δ is measured in kilograms per cubic meter. If we take the surface of the earth to be a sphere with radius 6370 km, then this model is a reasonable one for $6.370 \times 10^6 \leq \rho \leq 6.375 \times 10^6$. Use this model to estimate the mass of the atmosphere between the ground and an altitude of 5 km.

Solution: Replace this text with your solution. $\square$

Exercise (15.8.49)**.** The surfaces $\rho = 1 + \frac{1}{5}\sin m\theta \sin n\phi$ have been used as models for tumors. The "bumpy sphere" with $m = 6$ and $n = 5$ is shown. Use a computer algebra system to find the volume it encloses.



Solution: Replace this text with your solution. $\square$

Exercise (15.8.50)**.** Show that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{x^2+y^2+z^2}\,e^{-(x^2+y^2+z^2)}\,dx\,dy\,dz = 2\pi.$$

(The improper triple integral is defined as the limit of a triple integral over a solid sphere as the radius of the sphere increases indefinitely.)

Solution: Replace this text with your solution. □