St. Joseph's University
Date: November 3, 2025 Professor: Asher Roberts

Calculus III Midterm Exam

Number of questions—10

Directions: Solve each of the following problems using separate paper, while clearly indicating each problem number when solving. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 10 points. You must work alone, but you may use a graphing calculator as a supplement to your own work if you indicate the steps used. You may not use computational intelligence or AI.

1. Find an equation of the plane through the points $(2, 1, 2)$, $(3, -8, 6)$, and $(-2, -3, 1)$.
2. Find the length of the curve $\mathbf{r}(t) = t^2\,\mathbf{i} + 9t\,\mathbf{j} + 4t^{3/2}\,\mathbf{k}$, $1 \le t \le 4$.
3. Find the curvature of $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + e^t\,\mathbf{k}$.
4. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$ or show that it does not exist.
5. Find the first partial derivatives of $h(x, y, z, t) = x^2 y \cos(z/t)$.
6. Find the second partial derivatives of $v = \sin(s^2 - t^2)$.
7. Find $\partial z/\partial s$ and $\partial z/\partial t$ for $z = (\sin\theta)/r$, $r = st$, $\theta = s^2 + t^2$.
8. Find the directional derivative of $f(x, y, z) = x^2y + y^2z$ at the point $(1, 2, 3)$ in the direction of the vector $\mathbf{v} = \langle 2, -1, 2\rangle$.
9. Find the local maximum and minimum values and saddle point(s) of $f(x, y) = \sin x \sin y$, $-\pi < x < \pi$, $-\pi < y < \pi$.
10. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant c.