Linear Algebra Homework #1

Replace this text with your name

Due: Replace this text with a due date

Exercise (1.1.7)**.** In each part, find the augmented matrix for the linear system.

(a)

$$\begin{aligned} -2x_1 &= 6 \\ 3x_1 &= 8 \\ 9x_1 &= -3 \end{aligned}$$

(b)

$$\begin{aligned} 6x_1 - x_2 + 3x_3 &= 4 \\ 5x_2 - x_3 &= 1 \end{aligned}$$

(c)

$$\begin{aligned} 2x_2 \phantom{{}-x_3} - 3x_4 + x_5 &= 0 \\ -3x_1 - x_2 + x_3 \phantom{{}+2x_4-3x_5} &= -1 \\ 6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 &= 6 \end{aligned}$$

Solution: Replace this text with your solution. □

Exercise (1.1.9)**.** In each part, determine whether the given 3-tuple is a solution of the linear system

$$\begin{aligned} 2x_1 - 4x_2 - x_3 &= 1 \\ x_1 - 3x_2 + x_3 &= 1 \\ 3x_1 - 5x_2 - 3x_3 &= 1 \end{aligned}$$

(a) $(3, 1, 1)$

(b) $(3, -1, 1)$

(c) $(13, 5, 2)$

(d) $\left(\frac{13}{2}, \frac{5}{2}, 2\right)$

(e) $(17, 7, 5)$

Solution: Replace this text with your solution. □

Exercise (1.1.17)**.** In each part, find a single elementary row operation that will create a 1 in the upper left corner of the given augmented matrix and will not create any fractions in its first row.

(a)

$$\begin{bmatrix} -3 & -1 & 2 & 4 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$$

Solution: Replace this text with your solution. □

Exercise (1.2.7)**.** Solve the system by Gaussian elimination.

$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x - 3w &= -3 \end{aligned}$$

Solution: Replace this text with your solution. □

Exercise (1.2.27)**.** What condition, if any, must a, b, and c satisfy for the linear system to be consistent?

$$\begin{aligned} x + 3y - z &= a \\ x + y + 2z &= b \\ 2y - 3z &= c \end{aligned}$$

Solution: Replace this text with your solution. □

Exercise (1.2.29)**.** Solve the following system, where a and b are constants.

$$\begin{aligned} 2x + y &= a \\ 3x + 6y &= b \end{aligned}$$

Solution: Replace this text with your solution. □

Exercise (1.3.5)**.** Use the following matrices to compute the indicated expression if it is defined.

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

(a) AB

(b) BA

(c) $(3E)D$

(d) $(AB)C$

(e) $A(BC)$

(f) CC^T

(g) $(DA)^T$

(h) $(C^T B)A^T$

(i) $\mathrm{tr}(DD^T)$

(j) $\mathrm{tr}(4E^T - D)$

(k) $\mathrm{tr}(C^T A^T + 2E^T)$

(l) $\mathrm{tr}((EC^T)^T A)$

Solution: Replace this text with your solution. □

Exercise (1.3.15)**.** Find all values of k, if any, that satisfy the equation.

$$\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

Solution: Replace this text with your solution. □

Exercise (1.3.23)**.** Solve the matrix equation for a, b, c, and d.

$$\begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} d & d-2c \\ d+2c & -2 \end{bmatrix}$$

Solution: Replace this text with your solution. □

Exercise (1.4.9)**.** Find the inverse of

$$\begin{bmatrix} \frac{1}{2}(e^x + e^{-x}) & \frac{1}{2}(e^x - e^{-x}) \\ \frac{1}{2}(e^x - e^{-x}) & \frac{1}{2}(e^x + e^{-x}) \end{bmatrix}$$

Solution: Replace this text with your solution. □

Exercise (1.4.19)**.** Compute the following using the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

(a) A^3

(b) A^{-3}

(c) $A^2 - 2A + I$

Solution: Replace this text with your solution. □

Exercise (1.4.21)**.** Compute $p(A)$ for the following polynomials using the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

(a) $p(x) = x - 2$

(b) $p(x) = 2x^2 - x + 1$

(c) $p(x) = x^3 - 2x + 1$

Solution: Replace this text with your solution. □

Exercise (1.5.11)**.** Find the inverse of the matrix (if the inverse exists).

(a)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$$

Exercise (1.5.19)**.** Find the inverse of each of the following 4×4 matrices, where k_1, k_2, k_3, k_4, and k are all nonzero.

(a)

$$\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} k & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & k & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise (1.5.27)**.** Show that the matrices A and B are row equivalent by finding a sequence of elementary row operations that produces B from A, and then use that result to find a matrix C such that $CA = B$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & -2 \\ 1 & 1 & 4 \end{bmatrix}$$

Exercise (1.6.11)**.** Solve the linear systems. Using the given values for the b's solve the systems together by reducing an appropriate augmented matrix to reduced row echelon form.

$$\begin{aligned} 4x_1 - 7x_2 &= b_1 \\ x_1 + 2x_2 &= b_2 \end{aligned}$$

(i) $b_1 = 0$, $b_2 = 1$

(ii) $b_1 = -4$, $b_2 = 6$

(iii) $b_1 = -1$, $b_2 = 3$

(iv) $b_1 = -5$, $b_2 = 1$

Exercise (1.6.17)**.** Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.

$$\begin{aligned} x_1 - x_2 + 3x_3 + 2x_4 &= b_1 \\ -2x_1 + x_2 + 5x_3 + x_4 &= b_2 \\ -3x_1 + 2x_2 + 2x_3 - x_4 &= b_3 \\ 4x_1 - 3x_2 + x_3 + 3x_4 &= b_4 \end{aligned}$$

Exercise (1.6.19)**.** Solve the matrix equation for X.

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

Exercise (1.7.9)**.** Find A^2, A^{-2}, and A^{-k} (where k is any integer) by inspection.

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

Exercise (1.7.17)**.** Create a symmetric matrix by substituting appropriate numbers for the $\times$'s.

(a)

$$\begin{bmatrix} 2 & -1 \\ \times & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & \times & \times & \times \\ 3 & 1 & \times & \times \\ 7 & -8 & 0 & \times \\ 2 & -3 & 9 & 0 \end{bmatrix}$$

Exercise (1.7.27)**.** Find all values of x for which A is invertible.

$$A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$$

Exercise (1.8.13)**.** Find the standard matrix for the transformation T defined by the formula.

(a) $T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_2)$

(b) $T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, -x_1)$

(c) $T(x_1, x_2, x_3) = (0, 0, 0, 0, 0)$

(d) $T(x_1, x_2, x_3, x_4) = (x_4, x_1, x_3, x_2, x_1 - x_3)$

Exercise (1.8.27)**.** The images of the standard basis vectors for R^3 are given for a linear transformation $T : R^3 \to R^3$. Find the standard matrix for the transformation, and find $T(\mathbf{x})$.

$$T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \quad T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad T(\mathbf{e}_3) = \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Exercise (1.8.35)**.** Use matrix multiplication to find the orthogonal projection of $(-2, 1, 3)$ onto the

(a) xy-plane

(b) xz-plane

(c) yz-plane

Exercise (1.9.9)**.** Find the standard matrix for the stated composition in R^3.

(a) A reflection about the yz-plane, followed by an orthogonal projection onto the xz-plane.

(b) A reflection about the xy-plane, followed by an orthogonal projection onto the xy-plane.

(c) An orthogonal projection onto the xy-plane, followed by a reflection about the yz-plane.

Exercise (1.9.15)**.** Let $T_1 : R^2 \to R^4$ and $T_2 : R^4 \to R^3$ be given by:

$$T_1(x, y) = (y, x, x + y, x - y)$$
$$T_2(x, y, z, w) = (x + w, y + w, z + w).$$

(a) xy-plane

(b) xz-plane

(c) yz-plane

Exercise (1.9.19)**.** Determine whether the matrix operator $T : R^2 \to R^2$ defined by the equations is invertible; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2)$.

(a)

$$\begin{aligned} w_1 &= x_1 + 2x_2 \\ w_2 &= -x_1 + x_2 \end{aligned}$$

(b)

$$\begin{aligned} w_1 &= 4x_1 - 6x_2 \\ w_2 &= -2x_1 + 3x_2 \end{aligned}$$