Linear Algebra Homework #4

Replace this text with your name

Due: Replace this text with a due date

Exercise (4.1.11)**.** Show that the set of all pairs of real numbers of the form $(1, x)$ with the operations

$$(1, y) + (1, y') = (1, y + y') \quad \text{and} \quad k(1, y) = (1, ky)$$

is a vector space.

Solution: Replace this text with your solution. ☐

Exercise (4.1.17)**.** Show that the set of all points in R^2 lying on a line is a vector space with respect to the standard operations of vector addition and scalar multiplication if and only if the line passes through the origin.

Solution: Replace this text with your solution. ☐

Exercise (4.1.27)**.** Prove that the set M_{mn} of all $m \times n$ matrices with the usual operations of addition and scalar multiplication is a vector space.

Solution: Replace this text with your solution. ☐

Exercise (4.2.17)**.** Which of the following are subspaces of R^∞?

(a) All sequences of the form $\mathbf{v} = (v_1, v_2, \ldots, v_n, \ldots)$ such that $\lim_{n\to\infty} v_n = 0$.

(b) All convergent sequences (that is, all sequences of the form) $\mathbf{v} = (v_1, v_2, \ldots, v_n, \ldots)$ such that $\lim_{n\to\infty} v_n$ exists).

(c) All sequences of the form $\mathbf{v} = (v_1, v_2, \ldots, v_n, \ldots)$ such that $\sum_{n=1}^\infty v_n = 0$.

(d) All sequences of the form $\mathbf{v} = (v_1, v_2, \ldots, v_n, \ldots)$ such that $\sum_{n=1}^\infty v_n$ converges.

Solution: Replace this text with your solution. □

Exercise (4.2.19)**.** Determine whether the solution space of the system $A\mathbf{x} = \mathbf{0}$ is a line through the origin, or the origin only. If it is a plane, find an equation for it. If it is a line, find parametric equations for it.

(a) $A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 0 \\ 2 & -4 & -5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 4 \\ 3 & 1 & 11 \end{bmatrix}$

Solution: Replace this text with your solution. □

Exercise (4.2.21)**.** Show that the set of continuous functions $\mathbf{f} = f(x)$ on $[a, b]$ such that

$$\int_a^b f(x)\, dx = 0$$

is a subspace of $C[a, b]$.

Solution: Replace this text with your solution. □

Exercise (4.3.15)**.** Let W be the solution space to the system $A\mathbf{x} = \mathbf{0}$. Determine whether the set $\{\mathbf{u}, \mathbf{v}\}$ spans W.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) $\mathbf{u} = (1, 0, -1, 0)$, $\mathbf{v} = (0, 1, 0, -1)$

(b) $\mathbf{u} = (1, 0, -1, 0)$, $\mathbf{v} = (1, 1, -1, -1)$

Solution: Replace this text with your solution. □

Exercise (4.3.17)**.** In each part, let $T_A : R^2 \to R^2$ be multiplication by A, and let $\mathbf{u}_1 = (1, 2)$ and $\mathbf{u}_2 = (-1, 1)$. Determine whether the set $\{T_A(\mathbf{u}_1), T_A(\mathbf{u}_2)\}$ spans R^2.

(a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

Solution: Replace this text with your solution. □

Exercise (4.3.21)**.** Let V and W be subspaces of R^2 that are spanned by $(3, 1)$ and $(2, 1)$, respectively. Find a vector $\mathbf{v}$ in V and a vector $\mathbf{w}$ in W for which $\mathbf{v} + \mathbf{w} = (3, 5)$.

Solution: Replace this text with your solution. □

Exercise (4.4.9)**.** (a) Show that the three vectors $\mathbf{v}_1 = (0, 3, 1, -1)$, $\mathbf{v}_2 = (6, 0, 5, 1)$, and $\mathbf{v}_3 = (4, -7, 1, 3)$ form a linearly dependent set in R^4.

(b) Express each vector in part (a) as a linear combination of the other two.

Solution: Replace this text with your solution. □

Exercise (4.4.13)**.** In each part, let $T_A : R^2 \to R^2$ be multiplication by A, and let $\mathbf{u}_1 = (1, 2)$ and $\mathbf{u}_2 = (-1, 1)$. Determine whether the set $\{T_A(\mathbf{u}_1, T_A(\mathbf{u}_2)\}$ is linearly independent in R^2.

(a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

Solution: Replace this text with your solution. □

Exercise (4.4.21)**.** Use the Wronskian to show that the functions $f_1(x) = \sin x$, $f_2(x) = \cos x$, and $f_3(x) = x \cos x$ are linearly independent vectors in $C^\infty(-\infty, \infty)$.

Solution: Replace this text with your solution. □

Exercise (4.5.13)**.** Find the coordinate vector of $\mathbf{v}$ relative to the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for R^3.

(a) $\mathbf{v} = (2, -1, 3)$; $\mathbf{v}_1 = (1, 0, 0)$, $\mathbf{v}_2 = (2, 2, 0)$, $\mathbf{v}_3 = (3, 3, 3)$

(b) $\mathbf{v} = (5, -12, 3)$; $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (-4, 5, 6)$, $\mathbf{v}_3 = (7, -8, 9)$

Solution: Replace this text with your solution. □

Exercise (4.5.15)**.** First show that the set $S = \{A_1, A_2, A_3, A_4\}$ is a basis for M_{22}, then express A as a linear combination of the vectors in A, and then find the coordinate vector of A relative to S for

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

Solution: Replace this text with your solution. □

Exercise (4.5.21)**.** In each part, let $T_A : R^3 \to R^3$ be multiplication by A, and let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for R^3. Determine whether the set $\{T_A(\mathbf{e}_1), T_A(\mathbf{e}_2), T_A(\mathbf{e}_3)\}$ is linearly independent in R^2.

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

Solution: Replace this text with your solution. □

Exercise (4.6.15)**.** The vectors $\mathbf{v}_1 = (1, -2, 3)$ and $\mathbf{v}_2 = (0, 5, -3)$ are linearly independent. Enlarge $\{\mathbf{v}_1, \mathbf{v}_2\}$ to a basis for R^3.

Solution: Replace this text with your solution. $\square$

Exercise (4.6.17)**.** Find a basis for the subspace of R^3 that is spanned by the vectors

$$\mathbf{v}_1 = (1, 0, 0), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (2, 0, 1), \quad \mathbf{v}_4 = (0, 0, -1).$$

Solution: Replace this text with your solution. $\square$

Exercise (4.6.19)**.** In each part, let $T_A : R^3 \to R^3$ be multiplication by A and find the dimension of the subspace of R^3 consisting of all vectors $\mathbf{x}$ for which $T_A(\mathbf{x}) = \mathbf{0}$.

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Solution: Replace this text with your solution. $\square$

Exercise (4.7.9)**.** Let S be the standard basis for R^3, and let $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be the basis in which $\mathbf{v}_1 = (1, 2, 1)$, $\mathbf{v}_2 = (2, 5, 0)$, and $\mathbf{v}_3 = (3, 3, 8)$.

(a) Find the transition matrix $P_{B \to S}$ by inspection.

(b) Find the transition matrix $P_{S \to B}$.

(c) Confirm that $P_{B \to S}$ and $P_{S \to B}$ are inverses of one another.

(d) Let $\mathbf{w} = (5, -3, 1)$. Find $[\mathbf{w}]_B$ and then compute $[\mathbf{w}]_S$.

(e) Let $\mathbf{w} = (3, -5, 0)$. Find $[\mathbf{w}]_S$ and then compute $[\mathbf{w}]_B$.

Solution: Replace this text with your solution. □

Exercise (4.7.15)**.** Consider the matrix

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}.$$

(a) P is the transition matrix from what basis B to the standard basis $S = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ for R^3?

(b) P is the transition matrix from the standard basis $S = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ to what basis B for R^3?

Solution: Replace this text with your solution. □

Exercise (4.7.17)**.** Let $S = \{\mathbf{e}_1, \mathbf{e}_2\}$ be the standard basis for R^2, and let $B = \{\mathbf{v}_1, \mathbf{v}_2\}$ be the basis that results when the linear transformation defined by

$$T(x_1, x_2) = (2x_1 + 3x_2, 5x_1 - x_2)$$

is applied to each vector in S. Find the transition matrix $P_{B \to S}$.

Solution: Replace this text with your solution. □

Exercise (4.8.9)**.** Find bases for the null space and row space of A.

(a) $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

Solution: Replace this text with your solution. □

Exercise (4.8.17)**.** Find a subset of the vectors $\mathbf{v}_1 = (1,0,1,1)$, $\mathbf{v}_2 = (-3,3,7,1)$, $\mathbf{v}_3 = (-1,3,9,3)$, and $\mathbf{v}_4 = (-5,3,5,-1)$ that forms a basis for the space spanned by those vectors, and then express each vector that is not in the basis as a linear combination of the basis vectors.

Solution: Replace this text with your solution. □

Exercise (4.8.21)**.** In each part, let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix}$. For the given vector $\mathbf{b}$, find the general form of all vectors $\mathbf{x}$ in R^3 for which $T_A(\mathbf{x}) = \mathbf{b}$ if such vectors exist.

(a) $\mathbf{b} = (0,0)$

(b) $\mathbf{b} = (1,3)$

(c) $\mathbf{b} = (-1,1)$

Solution: Replace this text with your solution. □

Exercise (4.9.13)**.** Find the dimensions and bases for the four fundamental spaces of the matrix.

$$A = \begin{bmatrix} 0 & -1 & -4 \\ -1 & 0 & -4 \\ -2 & 3 & 4 \end{bmatrix}$$

Solution: Replace this text with your solution. □

Exercise (4.9.19)**.** Find bases for the four fundamental spaces of the matrix.

$$A = \begin{bmatrix} 0 & 2 & 8 & -7 \\ 2 & -2 & 4 & 0 \\ -3 & 4 & -2 & 5 \end{bmatrix}$$

Solution: Replace this text with your solution. □

Exercise (4.9.23)**.** Let $T : R^5 \to R^3$ be the linear transformation defined by the formula

$$T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2, x_2 + x_3 + x_4, x_4 + x_5).$$

(a) Find the rank of the standard matrix for T.

(b) Find the nullity of the standard matrix for T.

Solution: Replace this text with your solution. □