Linear Algebra Homework #6

Replace this text with your name

Due: Replace this text with a due date

Exercise (6.1.21)**.** Find $\|U\|$ and $d(U, V)$ relative to the standard inner product on M_{22} for

$$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}, \quad V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}.$$

Solution: Replace this text with your solution. □

Exercise (6.1.23)**.** Let

$$\mathbf{p} = x + x^3 \quad \text{and} \quad \mathbf{q} = 1 + x^2.$$

Find $\|\mathbf{p}\|$ and $d(\mathbf{p}, \mathbf{q})$ relative to the evaluation inner product on P_3 at the sample points $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, and $x_3 = 1$.

Solution: Replace this text with your solution. □

Exercise (6.1.33)**.** Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$. Show that the expression

$$\langle \mathbf{u}, \mathbf{v} \rangle = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$$

does *not* define an inner product on R^3, and list all inner product axioms that fail to hold.

Solution: Replace this text with your solution. □

Exercise (6.2.27)**.** Find a basis for the orthogonal complement of the subspace of R^n spanned by the vectors $\mathbf{v}_1 = (1, 4, 5, 2)$, $\mathbf{v}_2 = (2, 1, 3, 0)$, and $\mathbf{v}_3 = (-1, 3, 2, 2)$.

Solution: Replace this text with your solution. □

Exercise (6.2.33)**.** Let $C[-1, 1]$ have the integral inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)\, dx$$

and let $\mathbf{p} = p(x) = x^2 - x$ and $\mathbf{q} = q(x) = x + 1$.

(a) Find $\langle \mathbf{p}, \mathbf{q} \rangle$.

(b) Find $\|\mathbf{p}\|$ and $\|\mathbf{q}\|$.

Solution: Replace this text with your solution. □

Exercise (6.2.39)**.** Let $C[0, \pi]$ have the integral inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)\, dx$$

and let $\mathbf{f}_n = \cos nx$ $(n = 0, 1, 2, \ldots)$. Show that if $k \neq l$, then $\mathbf{f}_k$ and $\mathbf{f}_l$ are orthogonal vectors.

Solution: Replace this text with your solution. □

Exercise (6.3.29)**.** Let R^3 have the Euclidean inner product and use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ into an orthonormal basis where $\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (-1, 1, 0)$, and $\mathbf{u}_3 = (1, 2, 1)$.

Solution: Replace this text with your solution. $\square$

Exercise (6.3.43)**.** Let P_2 have the inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_0^1 p(x)q(x)\,dx.$$

Apply the Gram-Schmidt process to transform the standard basis $S = \{1, x, x^2\}$ into an orthonormal basis.

Solution: Replace this text with your solution. $\square$

Exercise (6.3.49)**.** Find a QR-decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}.$$

Solution: Replace this text with your solution. $\square$