St. Joseph's University
Date: October 29, 2025 Professor: Asher Roberts

Linear Algebra Midterm Exam

Number of questions—8

Directions: Solve each of the following problems using separate paper, while clearly indicating each problem number when solving. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 12.5 points. You must work alone, but you may use a graphing calculator as a supplement to your own work if you indicate the steps used. You may not use computational intelligence or AI.

1. Solve the system by Gaussian elimination.

$$\begin{aligned} 2x_1 - x_2 + 3x_3 + 4x_4 &= 9 \\ x_1 - 2x_3 + 7x_4 &= 11 \\ 3x_1 - 3x_2 + x_3 + 5x_4 &= 8 \\ 2x_1 + x_2 + 4x_3 + 4x_4 &= 10 \end{aligned}$$

2. Find the inverse of the matrix (if the inverse exists).

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$

3. Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= b_1 \\ 4x_1 - 5x_2 + 8x_3 &= b_2 \\ -3x_1 + 3x_2 - 3x_3 &= b_3 \end{aligned}$$

4. Determine whether the matrix operator $T : R^2 \to R^2$ defined by the equations is invertible; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2)$.

$$\begin{aligned} w_1 &= 2x_1 - 3x_2 \\ w_2 &= 5x_1 + x_2 \end{aligned}$$

5. Evaluate $\det(A)$ by a cofactor expansion along a row or column of your choice.

$$\begin{bmatrix} 1 & k & k^2 \\ 1 & k & k^2 \\ 1 & k & k^2 \end{bmatrix}$$

6. Find the adjoint of the matrix.

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

7. Find the vector component of $\mathbf{u}$ along $\mathbf{a}$ and the vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$ for $\mathbf{u} = (2, 0, 1)$, $\mathbf{a} = (1, 2, 3)$.

8. Determine whether $\mathbf{u} = (6, 3, 1)$, $\mathbf{v} = (-1, 1, 2)$, and $\mathbf{w} = (4, -3, -1)$ lie in the same plane when positioned so that their initial points coincide.