

Calculus III Optional Take-Home Midterm

Number of questions—10

Directions: Solve each of the following problems using separate paper, while clearly indicating each problem number when solving. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 10 points. You must work alone, but you may use a graphing calculator as a supplement to your own work if you indicate the steps used. You may not use a phone, computer, computational intelligence, AI, or other tools to assist you in solving the problems, nor may you have any of these devices in your possession while taking this exam.

1. Find an equation of the plane through the points $(3, 0, -1)$, $(-2, -2, 3)$, and $(7, 1, -4)$.
2. Find the length of the curve $\mathbf{r}(t) = \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}$, $0 \leq t \leq 1$.
3. Find the curvature of $\mathbf{r}(t) = \sqrt{6}t^2 \mathbf{i} + 2t \mathbf{j} + 2t^3 \mathbf{k}$.
4. Find $\lim_{(x,y) \rightarrow (1,1)} \frac{y-x}{1-y+\ln x}$ or show that it does not exist.
5. Find the first partial derivatives of $\phi(x, y, z, t) = \frac{\alpha x + \beta y^2}{\gamma z + \delta t^2}$.
6. Find the second partial derivatives of $z = \arctan \frac{x+y}{1-xy}$.
7. Find $\partial z / \partial s$ and $\partial z / \partial t$ for $z = \tan(u/v)$, $u = 2s + 3t$, $v = 3s - 2t$.
8. Find the directional derivative of $f(x, y, z) = xy^2 \tan^{-1} z$ at the point $(2, 1, 1)$ in the direction of the vector $\mathbf{v} = \langle 1, 1, 1 \rangle$.
9. Find the local maximum and minimum values and saddle point(s) of $f(x, y) = y^2 - 2y \cos x$, $-1 \leq x \leq 7$.
10. Find the dimensions of the rectangular box with largest volume if the total surface area is given as 64 cm^2 .