Elementary Number Theory Homework #1

Replace this text with your name

Due: Replace this text with a due date

Exercise (1.1.1)**.** Establish the formulas below by mathematical induction:

(a) $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ for all $n \geq 1$.

(b) $1 + 3 + 5 + \cdots + (2n-1) = n^2$ for all $n \geq 1$.

(c) $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$ for all $n \geq 1$.

(d) $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$ for all $n \geq 1$.

(e) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$ for all $n \geq 1$.

Solution: Replace this text with your solution. □

Exercise (1.1.4)**.** Prove that the cube of any integer can be written as the difference of two squares. [*Hint*: Notice that

$$n^3 = (1^3 + 2^3 + \cdots + n^3) - (1^3 + 2^3 + \cdots + (n-1)^3).]$$

Solution: Replace this text with your solution. □

Exercise (1.2.2)**.** If $2 \leq k \leq n-2$, show that

$$\binom{n}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k} \qquad n \geq 4.$$

Solution: Replace this text with your solution. □

Exercise (1.2.8)**.** Show that, for $n \geq 1$,

$$\binom{2n}{n} = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n} 2^{2n}.$$

Solution: Replace this text with your solution. □