Elementary Number Theory Homework #2

Replace this text with your name

Due: Replace this text with a due date

Exercise (2.1.4)**.** Prove that the square of any odd multiple of 3 is the difference of two triangular numbers; specifically, that

$$9(2n+1)^2 = t_{9n+4} - t_{3n+1}.$$

Solution: Replace this text with your solution. □

Exercise (2.1.8)**.** Prove that the sum of the reciprocals of the first n triangular numbers is less than 2; that is,

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \cdots + \frac{1}{t_n} < 2.$$

[*Hint:* Observe that $\frac{2}{n(n+1)} = 2(\frac{1}{n} - \frac{1}{n+1})$.]

Solution: Replace this text with your solution. □

Exercise (2.2.4)**.** Prove that $3a^2 - 1$ is never a perfect square.
[*Hint:* Use Example 4(a).]

Solution: Replace this text with your solution. □

Exercise (2.2.5)**.** For $n \geq 1$, prove that $n(n+1)(2n+1)/6$ is an integer.
[*Hint:* By the Division Algorithm, n has one of the forms $6k, 6k+1, \ldots, 6k+5$; establish the result in each of these six cases.]

Solution: Replace this text with your solution. □

Exercise (2.3.5)**.** Prove that for any integer a, one of the integers a, $a + 2$, $a + 4$ is divisible by 3.

Solution: Replace this text with your solution. $\square$

Exercise (2.3.7)**.** Prove that if a and b are both odd integers, then $16 \mid a^4 + b^4 - 2$.

Solution: Replace this text with your solution. $\square$

Exercise (2.4.5)**.** For $n \geq 1$, and positive integers a, b, show the following:

(a) If $\gcd(a, b) = 1$, then $\gcd(a^n, b^n) = 1$.
[*Hint:* Use the fact that if $\gcd(a, b) = 1$, and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$.]

(b) The relation $a^n \mid b^n$ implies that $a \mid b$.
[*Hint:* Put $d = \gcd(a, b)$ and write $a = rd$, $b = sd$, where $\gcd(r, s) = 1$. By part (a), $\gcd(r^n, s^n) = 1$. Show that $r = 1$, whence $a = d$.]

Solution: Replace this text with your solution. □

Exercise (2.4.6)**.** Prove that if $\gcd(a, b) = 1$, then $\gcd(a + b, ab) = 1$.

Solution: Replace this text with your solution. □

Exercise (2.5.2)**.** Determine all solutions in the integers of the following Diophantine equations:

(a) $56x + 72y = 40$.

(b) $24x + 138y = 18$.

(c) $221x + 35y = 11$.

Solution: Replace this text with your solution. $\square$

Exercise (2.5.6)**.** A farmer purchased 100 head of livestock for a total cost of \$4000. Prices were as follow: calves, \$120 each; lambs, \$50 each; piglets, \$25 each. If the farmer obtained at least one animal of each type, how many of each did he buy?

Solution: Replace this text with your solution. $\square$