

## Elementary Number Theory Homework #3

Replace this text with your name

Due: Replace this text with a due date

**Exercise (3.1.10).** If  $p \neq 5$  is an odd prime, prove that either  $p^2 - 1$  or  $p^2 + 1$  is divisible by 10.

*Solution:* Replace this text with your solution. □

**Exercise (3.1.13).** If  $n > 1$  is an integer not of the form  $6k + 3$ , prove that  $n^2 + 2^n$  is composite.

[*Hint:* Show that either 2 or 3 divides  $n^2 + 2^n$ .]

*Solution:* Replace this text with your solution. □

**Exercise (3.2.5).** Show any composite three-digit number must have a prime factor less than or equal to 31.

*Solution:* Replace this text with your solution. □

**Exercise (3.2.8).** Give another proof of the infinitude of primes by assuming that there are only finitely many primes, say  $p_1, p_2, \dots, p_n$ , and using the following integer to arrive at a contradiction:

$$N = p_2 p_3 \cdots p_n + p_1 p_3 \cdots p_n + \cdots + p_1 p_2 \cdots p_{n-1}.$$

*Solution:* Replace this text with your solution. □

**Exercise (3.3.12).** Let  $p_n$  denote the  $n$ th prime number. For  $n \geq 3$ , prove that  $p_{n+3}^2 < p_n p_{n+1} p_{n+2}$ .  
[Hint: Note that  $p_{n+3}^2 < 4p_{n+2}^2 < 8p_{n+1} p_{n+2}$ .]

*Solution:* Replace this text with your solution. □

**Exercise (3.3.20).** If  $p$  and  $p^2 + 8$  are both prime numbers, prove that  $p^3 + 4$  is also prime.

*Solution:* Replace this text with your solution. □