Elementary Number Theory Homework #4

Replace this text with your name

Due: Replace this text with a due date

Exercise (4.2.3)**.** If $a \equiv b \pmod{n}$, prove that $\gcd(a, n) = \gcd(b, n)$.

Solution: Replace this text with your solution. □

Exercise (4.2.13)**.** Verify that if $a \equiv b \pmod{n_1}$ and $a \equiv b \pmod{n_2}$, then $a \equiv b \pmod{n}$, where the integer $n = \operatorname{lcm}(n_1, n_2)$. Hence, whenever n_1 and n_2 are relatively prime, $a \equiv b \pmod{n_1 n_2}$.

Solution: Replace this text with your solution. □

Exercise (4.3.3)**.** Find the last two digits of the number 9^{9^9}.

Solution: Replace this text with your solution. □

Exercise (4.3.10)**.** Prove that no integer whose digits add up to 15 can be a square or a cube.
[*Hint:* For any a, $a^3 \equiv 0$, 1, or 8 (mod 9).]

Solution: Replace this text with your solution. □

Exercise (4.4.3)**.** Find all solutions of the linear congruence $3x - 7y \equiv 11 \pmod{13}$.

Solution: Replace this text with your solution. □

Exercise (4.4.14)**.** A certain integer between 1 and 1200 leaves the remainders 1, 2, 6 when divided by 9, 11, 13, respectively. What is the integer?

Solution: Replace this text with your solution. □