Elementary Number Theory Homework #5

Replace this text with your name

Due: Replace this text with a due date

Exercise (5.2.3)**.** From Fermat's theorem deduce that, for any integer $n \geq 0$, $13 \mid 11^{12n+6} + 1$.

Solution: Replace this text with your solution. $\square$

Exercise (5.2.5)**.** If $\gcd(a, 30) = 1$, show that 60 divides $a^4 + 59$.

Solution: Replace this text with your solution. $\square$

Exercise (5.3.2)**.** Determine whether 17 is a prime by deciding whether $16! \equiv -1 \pmod{17}$.

Solution: Replace this text with your solution. □

Exercise (5.3.15)**.** Verify that $4(29!) + 5!$ is divisible by 31.

Solution: Replace this text with your solution. □

Exercise (5.4.2)**.** Prove that a perfect square must end in one of the following pairs of digits: 00, 01, 04, 09, 16, 21, 24, 25, 29, 36, 41, 44, 49, 56, 61, 64, 69, 76, 81, 84, 89, 96.
[*Hint:* Because $x^2 \equiv (50 + x)^2 \pmod{100}$ and $x^2 \equiv (50 - x)^2 \pmod{100}$, it suffices to examine the final digits of x^2 for the 26 values $x = 0, 1, 2, \ldots, 25$.]

Solution: Replace this text with your solution. □

Exercise (5.4.6)**.** Factor 13561 with the help of the congruences

$$233^2 \equiv 3^2 \cdot 5 \pmod{13561} \qquad \text{and} \qquad 1281^2 \equiv 2^4 \cdot 5 \pmod{13561}.$$

Solution: Replace this text with your solution. □