Elementary Number Theory Homework #9

Replace this text with your name

Due: Replace this text with a due date

Exercise (9.1.1)**.** Solve the following quadratic congruences:

(a) $x^2 + 7x + 10 \equiv 0 \pmod{11}$.

(b) $3x^2 + 9x + 7 \equiv 0 \pmod{13}$.

(c) $5x^2 + 6x + 1 \equiv 0 \pmod{23}$.

Solution: Replace this text with your solution. □

Exercise (9.1.4)**.** Show that 3 is a quadratic residue of 23, but a nonresidue of 31.

Solution: Replace this text with your solution. □

Exercise (9.2.5)**.** Prove that 2 is not a primitive root of any prime of the form $p = 3 \cdot 2^n + 1$, except when $p = 13$.
[*Hint:* Use Theorem 9.2.5.]

Solution: Replace this text with your solution. □

Exercise (9.2.7)**.** If p is an odd prime, show that

$$\sum_{a=1}^{p-2} (a(a+1)/p) = -1.$$

[*Hint:* If a' is defined by $aa' \equiv 1 \pmod{p}$, then $(a(a+1)/p) = ((1+a')/p)$. Note that $1 + a'$ runs through a complete set of residues modulo p, except for the integer 1.]

Solution: Replace this text with your solution. □

Exercise (9.3.2)**.** Prove that 3 is a quadratic nonresidue of all primes of the form $2^{2n}+1$ and also all primes of the form 2^p-1 where p is an odd prime. [*Hint:* For all n, $4^n \equiv 4 \pmod{12}$.]

Solution: Replace this text with your solution. □

Exercise (9.3.4)**.** Verify that if p is an odd prime, then

$$(-2/p) = \begin{cases} 1 & \text{if } p \equiv 1 \pmod 8 \text{ or } p \equiv 3 \pmod 8 \\ -1 & \text{if } p \equiv 5 \pmod 8 \text{ or } p \equiv 7 \pmod 8. \end{cases}$$

Solution: Replace this text with your solution. □

Exercise (9.4.3)**.** Solve the congruence $x^2 \equiv 31 \pmod{11^4}$.

Solution: Replace this text with your solution. □

Exercise (9.4.4)**.** Find the solutions of $x^2 + 5x + 6 \equiv 0 \pmod{5^3}$ and $x^2 + x + 3 \equiv 0 \pmod{3^3}$.

Solution: Replace this text with your solution. □