St. Joseph's University
Date: March 4, 2026 Professor: Asher Roberts

Number Theory Midterm Exam

Number of questions—4

Directions: Solve each of the following problems using separate paper, while clearly indicating each problem number when solving. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 25 points. You must work alone, but you may refer to printed materials, as long as you do not share them. You may not use a calculator, phone, computer, computational intelligence, AI, or other tools to assist you in solving the problems.

1. Prove that
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)n}=1-\frac{1}{n}$$
for $n=2,3,\ldots$.

2. A number theory class consists of sophomores, juniors, and seniors. If each sophomore contributes \$125, each junior \$90, and each senior \$50, the professor will have \$2500 to treat himself to a cruise. There are 26 students; how many of each?

3. Prove that n^3-n is divisible by 6 for $n=2,3,\ldots$.

4. Let p be the smallest prime factor of n, where n is composite. Prove that if $p>n^{1/3}$, then n/p is prime.