Real Analysis Homework #4

Replace this text with your name

Due: Replace this text with a due date

Exercise (4.2.5)**.** Supply a proof for the following limit statements.

(a) $\lim_{x\to 2}(3x+4) = 10$.

(b) $\lim_{x\to 0} x^3 = 0$.

(c) $\lim_{x\to 2}(x^2+x-1) = 5$.

(d) $\lim_{x\to 3} 1/x = 1/3$.

Solution: Replace this text with your solution. □

Exercise (4.2.7)**.** Let $g : A \to \mathbf{R}$ and assume that f is a bounded function on A in the sense that there exists $M > 0$ satisfying $|f(x)| \leq M$ for all $x \in A$.

Show that if $\lim_{x\to c} g(x) = 0$, then $\lim_{x\to c} g(x)f(x) = 0$ as well.

Solution: Replace this text with your solution. □

Exercise (4.3.1)**.** Let $g(x) = \sqrt[3]{x}$.

(a) Prove that g is continuous at $c = 0$.

(b) Prove that g is continuous at a point $c \neq 0$. (The identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ will be helpful.)

Solution: Replace this text with your solution. □

Exercise (4.3.9)**.** Assume $h : \mathbf{R} \to \mathbf{R}$ is continuous on $\mathbf{R}$ and let $K = \{x : h(x) = 0\}$. Show that K is a closed set.

Solution: Replace this text with your solution. □

Exercise (4.4.5)**.** Assume that g is defined on an open interval (a, c) and it is known to be uniformly continuous on $(a, b]$ and $[b, c)$, where $a < b < c$. Prove that g is uniformly continuous on (a, c).

Solution: Replace this text with your solution. □

Exercise (4.4.11)**. (Topological Characterization of Continuity).** Let g be defined on all of $\mathbf{R}$. If B is a subset of $\mathbf{R}$, define the set $g^{-1}(B)$ by

$$g^{-1}(B) = \{x \in \mathbf{R} : g(x) \in B\}.$$

Show that g is continuous if and only if $g^{-1}(O)$ is open whenever $O \subseteq \mathbf{R}$ is an open set.

Solution: Replace this text with your solution. □

Exercise (4.5.7)**.** Let f be a continuous function on the closed interval $[0, 1]$ with range also contained in $[0, 1]$. Prove that f must have a fixed point; that is, show $f(x) = x$ for at least one value of $x \in [0, 1]$.

Solution: Replace this text with your solution. □

Exercise (4.5.8)**. (Inverse functions).** If a function $f : A \to \mathbf{R}$ is one-to-one, then we can define the inverse function f^{-1} on the range of f in the natural way: $f^{-1}(y) = x$ where $y = f(x)$.

Show that if f is continuous on an interval $[a, b]$ and one-to-one, then f^{-1} is also continuous.

Solution: Replace this text with your solution. □