St. Joseph's University
Date: March 9, 2026 Professor: Asher Roberts

Real Analysis Midterm Exam

Number of questions—4

Directions: Solve each of the following problems using separate paper, while clearly indicating each problem number when solving. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Each problem is worth 25 points. You must work alone, but you may refer to printed materials, as long as you do not share them. You may not use a calculator, phone, computer, computational intelligence, AI, or other tools to assist you in solving the problems.

1. Prove that $\inf S = -\sup\{-s : s \in S\}$, where $S \subseteq \mathbf{R}$ is nonempty and bounded below.
2. Prove that $\lim(c/n) = 0$ for any $c \in \mathbf{R}$.
3. Let (x_n) be a Cauchy sequence such that $x_n \in \mathbf{N}$ for every $n \in \mathbf{N}$. Show that (x_n) is eventually constant.
4. Suppose $\sum a_n$ with $a_n > 0$ is convergent, and $b_n = (a_1 + \cdots + a_n)/n$ for any $n \in \mathbf{N}$. Determine whether $\sum b_n$ converges.