

# Statistics Optional Take-Home Test 3

## Number of questions—4

**Directions:** Solve each of the following problems using separate paper, while clearly indicating each problem number when solving. Irrelevant work will detract from your score, while answers without work shown will be awarded no credit. Answers with partially correct work will receive partial credit. Unless otherwise specified, round all answers to 3 decimal places when necessary. You must work alone, but you may use a graphing calculator and the provided appendix tables as a supplement to your own work if you indicate the steps used. You may not use a phone, computer, computational intelligence, AI, or other tools to assist you in solving the problems.

1. Aldrich Ames is a convicted traitor who leaked American secrets to a foreign power. Yet Ames took routine lie detector tests and each time passed them. How can this be done? Recognizing control questions, employing unusual breathing patterns, biting one's tongue at the right time, pressing one's toes hard to the floor, and counting backward by 7 are countermeasures that are difficult to detect but can change the results of a polygraph examination (Source: *Lies! Lies!! Lies!!! The Psychology of Deceit*, by C. V. Ford, professor of psychiatry, University of Alabama). In fact, it is reported in Professor Ford's book that after only 20 minutes of instruction by "Buzz" Fay (a prison inmate), 85% of those trained were able to pass the polygraph examination even when guilty of a crime. Suppose that a random sample of nine students (in a psychology laboratory) are told a "secret" and then given instructions on how to pass the polygraph examination without revealing their knowledge of the secret.
  - (a) (6 pts) What are the  $n$ ,  $p$ ,  $q$ , and  $r$  that can be used to determine the probability that all the students are able to pass the polygraph examination?
  - (b) (6 pts) What is the probability that all the students are able to pass the polygraph examination?
  - (c) (6 pts) What is the probability that three or fewer of the students are able to pass the polygraph examination?
  - (d) (6 pts) What is the expected number of students able to pass the polygraph examination?
  - (e) (6 pts) What is the standard deviation of the distribution of the number of students able to pass the polygraph examination?
2. A relay microchip in a telecommunications satellite has a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months. When this computer-relay microchip malfunctions, the entire satellite is useless. A large London insurance company is going to insure the satellite for \$50 million. Assume that

the only part of the satellite in question is the microchip. All other components will work indefinitely.

- (a) (5 pts) What  $z$ -value corresponds to 84 months?
  - (b) (5 pts) If the satellite is insured for 84 months, what is the probability that it will malfunction before the insurance coverage ends?
  - (c) (5 pts) 99% of the area under the standard normal distribution lies to the left of what  $z$ -value?
  - (d) (5 pts) For how many months should the satellite be insured for the probability to be 99% that it will last beyond the insurance date?
3. Let  $x$  represent the dollar amount spent on supermarket impulse buying in a 10-minute (unplanned) shopping interval. Based on a *Denver Post* article, the mean of the  $x$  distribution is about \$20 and the estimated standard deviation is about \$7.
- (a) (5 pts) Consider a random sample of  $n = 100$  customers, each of whom has 10 minutes of unplanned shopping time in a supermarket. From the central limit theorem, what can you say about the probability distribution of  $\bar{x}$ , the average amount spent by these customers due to impulse buying?
  - (b) (5 pts) What are the mean and standard deviation of the  $\bar{x}$  distribution?
  - (c) (5 pts) What are the  $z$ -values corresponding to  $\bar{x}$  values of \$18 and \$22?
  - (d) (5 pts) What is the probability that  $\bar{x}$  is between \$18 and \$22?
  - (e) (5 pts) Let us assume that  $x$  has a distribution that is approximately normal. What are the  $z$ -values corresponding to  $x$  values of \$18 and \$22?
  - (f) (5 pts) What is the probability that  $x$  is between \$18 and \$22? (Again, assume that  $x$  has a distribution that is approximately normal.)
4. Over the past several months, an adult patient has been treated for tetany (severe muscle spasms). This condition is associated with an average total calcium level below 6 mg/dl (Reference: *Manual of Laboratory and Diagnostic Tests* by F. Fischbach). Recently, the patient's total calcium tests gave 35 readings with mean  $\bar{x} = 9.95$  and  $s = 1.02$ .
- (a) (5 pts) What distribution would be appropriate to use to compute a confidence interval for the population mean  $\mu$ ? Explain your answer.
  - (b) (5 pts) Find the critical value for a 99.9% confidence level.
  - (c) (5 pts) Find a 99.9% confidence interval for  $\mu$ .
  - (d) (5 pts) What does the confidence interval tell us in the context of the problem?